

Knots and Crosses

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Introduction

Tic Tac Toe is a board game in which we put crosses and knots in a 3X3 grid. The first to put three crosses or knots in a vertical, horizontal, or diagonal row wins.

The origin of Tic Tac Toe came from Ancient Egypt. The game of Tic Tac Toe then is more similar to the one that was first played in Ancient Rome, called "Termin Lapilli". Later, in the 18th century in England, a children's game much more similar to Tic Tac Toe appeared, which is the current game we are playing now.

1.1 Aim

The aim of this project is to research, understand and find the best and easiest tips and tricks to solve the different grid variants of tic-tac-toe, namely, of the 3X3, 4X4 and the 5X5 grid.

1.2.1 Objectives of this project

To investigate on:

- the strategies to win Tic Tac Toe
- how Mathematics is used in Tic Tac Toe
- how to win or tie with opponent in Tic Tac Toe without losing

1.2.2 Research Questions

Research question 1

How do we apply Mathematics (using probabilities or percentage of chance) to win Tic Tac Toe?

Research question 2

What are the strategies to win Tic Tac Toe in a 3x3 grid?

Research question 3

What are the strategies to win Tic Tac Toe in a 4x4 grid?

Research question 4

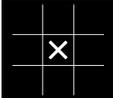
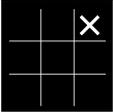
What are the strategies to win Tic Tac Toe in a 5x5 grid?

Method and Results

2.1 Table of basic strategies to win 3X3, 4X4 and 5X5

My team and I have found out that 3X3 is impossible to win if both players know where to play. In the end, it would be a tie.

General Table

-	3X3 (placing in middle) 	3X3 (placing at corner) 	4X4 (for whole board)	5X5
Total Matches	100	100	100	100
Wins	38	32	98	99
Losses	4	0	0	0
Tie	58	68	2	1
Win Rate	38%	32%	98%	99%

Strategies for 3X3

For 3X3, if you place in the center and the opponent picks the corner next, four out of six position will end with you winning, and that is a fraction of:

$$\frac{2}{3}$$

Strategies for 4X4

For 4X4, if you place in any of the four centers (R2 C2, R2 C3, R3 C2, R3 C3), there is a higher chance of winning. This is because if you start from the middle, you have at least 7 places to put your next piece (the opponent might place next to you). This gives you an 87.5% chance of winning ($7/8$) letting you win very easily. If you do not understand what I am talking about, the strategies are shown below.

If you start from the middle (R2 C2, R2 C3, R3 C2, R3 C3):

	C1	C2	C3	C4
R1				
R2		O	X	
R3				
R4				

	C1	C2	C3	C4
R1				
R2		O	X	
R3			X	
R4				

	C1	C2	C3	C4
R1				
R2		O	X	
R3			X	
R4			O	

	C1	C2	C3	C4
R1			X	
R2		O	X	
R3			X	
R4			O	

(Fig 1)

(Fig 2)

(Fig 3)

(Fig 4)

If you start from any of the middle (as shown in Fig 1), it would look like this. Your opponent would most probably put his/her knot next to you in an attempt to stop you from getting 3 in a row. If this happens, you can put your cross on the opposite side (For example, if the opponent puts his/her move horizontally, you can put it vertically and vice versa. If the opponent puts it diagonally, you can put it either horizontally or vertically.) If this does not happen (the opponent places his/her knot somewhere else), you can put it anywhere you want, as long as it is next to your first cross. This allows you to have 2 in a row, giving you a

100% chance of winning. Even if your opponent tried stopping you(as shown in Fig 3), you still have the other side to place your cross(as shown in Fig 4), letting you win.

If you start from the sides(not including the corners)(R1 C2, R1 C3, R2 C1, R2 C4, R3 C1, R3 C4, R4 C2, R4 C3,):

	C1	C2	C3	C4
R1		○	X	
R2				
R3				
R4				

(Fig 5)

	C1	C2	C3	C4
R1		○	X	
R2		X		
R3	○			
R4				

(Fig 6)

	C1	C2	C3	C4
R1		○	X	
R2	○	X	X	
R3	○			
R4				

(Fig 7)

	C1	C2	C3	C4
R1		○	X	
R2	○	X	X	
R3	○		X	
R4				

(Fig 8)

If you start from the sides(excluding the corners)(as shown in Fig 5), you can still win the game of Tic Tac Toe, it is slightly harder than starting in the middle but still manageable, if you follow these strategies. The opponent would try to stop you by placing his/her knot next to your cross(as shown in Fig 5). For your next move, you can put your cross diagonally, as long as it is diagonal to your first cross. You have created your first 2 in a row, but it is not your real method to win(it is to trick your opponent). Your opponent would try to block you by placing his knot at the end of your 2 crosses to try to stop you from getting 3 in a row(as shown in Fig 6). How, you just need to place 2 more crosses to win. Place your next cross at the point where the 2 crosses(placed earlier) met(as shown in Fig 7). Now, you will have 2 spots to place your next cross, ensuring you a 100% win. Your opponent tries to stop you by placing his/her knot at one end of the 2 crosses(as shown in Fig 7). Just place your next cross at the other end, and you can win the game(as shown in Fig 8).

If you start from the corners(R1 C1, R1 C4, R4 C1, R4 C4):

Method 1

	C1	C2	C3	C4
R1	X		X	
R2	○			
R3				
R4				

(Fig 8)

	C1	C2	C3	C4
R1	X		X	
R2	○	X		
R3				
R4				

(Fig 9)

	C1	C2	C3	C4
R1	X	X	X	
R2	○	X		
R3			○	
R4				

(Fig 10)

	C1	C2	C3	C4
R1	X	X	○	
R2	○	X		
R3		X	○	
R4				

(Fig 11)

Method 2:

	C1	C2	C3	C4
R1	X		X	
R2		○		
R3				
R4				

(Fig 12)

	C1	C2	C3	C4
R1	X		X	
R2	X	○		
R3	○			
R4				

(Fig 13)

	C1	C2	C3	C4
R1	X		X	
R2	X	○		
R3	○			
R4				

(Fig 14)

	C1	C2	C3	C4
R1	X	○	X	
R2	X	○		
R3	○			
R4				

(Fig 15)

	C1	C2	C3	C4
R1	X	○	X	
R2	X	○		
R3	○	X		
R4				

(Fig 16)

	C1	C2	C3	C4
R1	X	○	X	
R2	X	○		
R3	○	X		
R4			○	

(Fig 17)

	C1	C2	C3	C4
R1	X	○	X	
R2	X	○	X	
R3	○	X		
R4			○	

(Fig 18)

	C1	C2	C3	C4
R1	X	○	X	X
R2	X	○	X	
R3	○	X	○	
R4			○	

(Fig 19)

Normally, people would not start from the corners as the chances of winning are very low. You only have 3 directions to move towards. If your opponent places near your cross, you only have two directions left to place your next cross. However, if you insist on placing in the corners, the strategies are below.

Method 1: When you start placing in the corner, your opponent would place his/her knot close to your cross, and in method 1, your opponent puts his/her cross vertically to your cross(as shown in Fig 8). To try to “break free”, you can move towards the centre by placing your cross diagonally, stretching towards the centre so you can have more spots to place your crosses(as shown in Fig 9). Now, you have formed 2 crosses in a row. To stop you from getting 3 in a row, your opponent would place his/her knot at the end of your 2 crosses. Just place your next knot at the spot where the first 2 crosses meet(as shown in Fig

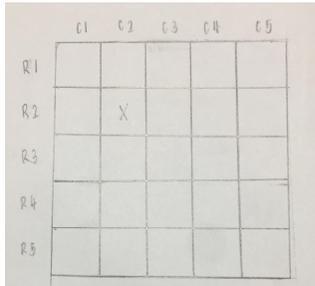
10). You will now have 2 directions to get 3 in a row. Even if your opponent tries to stop you by blocking one of your “2-in-a-rows”, you can place your next cross at the end of the other 2 crosses to win(as shown in Fig 11).

Method 2: In the first method, your opponent places his/her knot vertically to your cross. However, what if your opponent places his knot diagonally to your cross(as shown in Fig 12)? Don't worry, there is still a strategy for this. First, extend your crosses outwards so you have more directions to move towards. You cannot move towards the centre, because your opponent is blocking you. Therefore, I chose to move downwards by placing my next cross vertically to my first cross. This would form 2 crosses in a row. Your opponent would try to prevent you from getting 3 in a row by placing his knot at the end of your 2 crosses(as shown in Fig 13). Now, your opponent has created 2 knots in a row(as shown in Fig 13). To stop him, you can place your next cross at the end of his/her 2 knots(as shown in Fig 14). Your opponent would place his/her knot in between your 2 crosses to prevent you from getting 3 in a row. This would make your opponent form 2 knots in a row(as shown in Fig 15). To stop him from getting 3 knots in a row, place your cross at the end of his 2 knots. You have now created 2 crosses in a row(as shown in Fig 16). Your opponent tries to stop you from getting 3 in a row by placing his knot at the end of your 2 crosses(as shown in Fig 17). This is your final step. Place your next cross at R2 C3. This allows you to have 3 directions to form 3 in a row to win the game(as shown in Fig 18). Even if your opponent tries to stop you, it is impossible to do so. You can place your cross at any of the 2 ends your opponent did not block, giving you a 3 in a row and winning the game(as shown in Fig 19).

Strategies for 5X5

If you have the start

If you have the start as X, it is best to place your first move at:



(Fig. 20)

(Coordinates: R2, C2, or any of the other 3 corners of the middle layer of the grid)

This is to bait/trick the adversary to continue at R1, C1 or R2, C3. After the enemy places their move at either of these positions, you will have a huge advantage over your opponent as you can plan various attack routes to win (will be explained in the next strategy).

Please refrain from starting in the middle because this makes space for your opponent to plan more attacks to win over you.

If you don't have the start

Say your opponent has the start as X, and you are O.

	C1	C2	C3	C4	C5
R1	X1				
R2					
R3					
R4					
R5					

(Fig. 21)

If X starts at R1, C1 or the corners of the outermost layer of the grid (R1, C5 R5, C1 R5, C5),

	C1	C2	C3	C4	C5
R1	X1				
R2		O1			O1
R3					
R4					
R5					

(Fig. 22)

O will need to continue at R2, C2 or R2, C5 to attack X.

	C1	C2	C3	C4	C5
R1	X1				
R2		O1	X2		O1
R3					
R4					
R5					

(Fig. 23)

X will then be baited to place his move at R2, C3.

X needs to place his move at R2, C3 (X2) because they need to defend other first move (X1).

	C1	C2	C3	C4	C5
R1	X1				
R2		O1	X2		O1
R3					
R4					
R5		O2			O2

(Fig. 24)

Now, O has 3 attack routes.

O has a higher advantage to X now, because:

- X can defend one of the 3 attacks, and O can expand with the other 2 attacks
- By the time X blocks one of these 2 attacks, O would have completed the other

These factors will result in X having a slow defeat.

3 Conclusion

- In a 3X3 grid, the most effective strategy to win Tic Tac Toe is by placing in the middle and placing at the corner.
- In a 4X4 grid, the person who starts first always wins if you know the strategy, unless you place the cross/knot in the wrong spot. The easiest and most effective strategy to win Tic Tac Toe is by starting in the middle of the board(Row 2 Column 2, Row 3 Column 2, Row 2 Column 3, Row 3 Column 3). This would allow you to have at least 7 spots to place your next piece, giving you a 100% chance of winning. Although you can still start from the other parts of the grid, the chances of winning gets lower as you approach the tighter spaces(corners or sides) because you have lesser directions to move towards. There would also be a lower chance of winning because your opponent can prevent you from getting 3 in a row more easily. Strategies are very important in games like Tic Tac Toe. These games require thinking and finding out the most efficient way of winning. In order to get strategies, we sometimes need to use Mathematics to calculate the best route to win. If not, your strategy might not work and you might lose. Therefore, Mathematics is very important in our lives because it makes our lives easier.
- In a 5X5 grid, the most effective strategy to win Tic Tac Toe is to start in the corners of the middle layer of the grid. This will allow for more space to place your next moves for greater advantage.

4 References

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