PROJECT WORK WRITTEN REPORT
CATEGORY 8

CORNERING CORNERS
GROUP 8-30

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1. Introduction

In land-scarce Singapore, our buildings are becoming taller and living spaces smaller (Figure 1). This makes moving items around tight corners increasingly difficult which is a common problem faced by movers.

HOUSEHOLD SIZE VERSUS FLAT SIZE AND LIVING SPACE PER PERSON

Period	Flat type	(a) Floor area	(b) Average household size	Living space per person = (a)/(b)
1980s	3-room	69 sq m	4.6	15 sq m
	4-room	105 sq m		23 sq m
	5-room	123 sq m		27 sq m
	Executive	145 sq m		32 sq m
1990s	3-room	n/a	3.9	n/a
	4-room	100 sq m		26 sq m
	5-room	120 sq m		31 sq m
	Executive	140 sq m		36 sq m
2000s	3-room	65 sq m	3.4	19 sq m
	4-room	90 sq m		26 sq m
	5-room	110 sq m		32 sq m
	Executive	n/a		n/a

Source: HDB

ST GRAPHICS

Figure 1 - Household Size versus Flat Size and Living Space per Person

1.1 Objectives

Despite research done on similar topics with the “moving-sofa problem” being the most prominent. The solutions are often too difficult for the amateur to understand and apply.

Hence, our project aims to:

- simplify solutions to this problem by finding a more generic formula to move n-sided / irregular shapes around corners,
- create a program to assist consumers.

We hope to reduce problems faced by movers and help consumers to buy the correct-sized furniture for their homes.

1.2 Research Problems

1.2.1 Research Statements

(a) To investigate the formula/e to move *n-sided* shapes around corners for right-angle, acute-angle, obtuse-angle, for:

(i) $n = 3$

(ii) $n = 4$

(iii) $n = \infty$

(b) To investigate the formula/e to move irregular shapes around a π/2 corridor.

1.2.2 Simulation / Computer Programming

(a) Use Geogebra to animate the problem.

(b) Use Python to create a programme to determine if a shape can pass around a corner.

2. Literature Review

Retrieved From:

Josić, K. J. (n.d.). *No. 3234: The Moving Sofa Problem*. www.Uh.Edu. Retrieved August 13, 2021, from https://www.uh.edu/engines/epi3234.htm

This website states the history of the Moving Sofa Problem. The moving sofa problem came from over 50 years ago when the Canadian mathematician Leo Moser asked what is the shape of the largest possible sofa that can pass through the hallway. Mathematicians have found increasingly bigger sofas that do the job but we still don't know the shape of the biggest sofa that does the trick. However, instead of finding the biggest sofa possible, we intend to find a set of conditions such that if a shape satisfies all the conditions, it can pass through the hallway.

Retrieved From:

Boute, R. T. B. (2004, May). *Moving a Rectangle around a Corner—Geometrically*. https://biblio.ugent.be/publication/314475/file/452143.pdf

This paper is similar to our part on different widths and it helped with our part a lot by showing us a new perspective for approaching our problem. However, the paper doesn't show much working.

Retrieved From:

Romik, D. R., & Kallus, Y. K. (2018, October). *Improved upper bounds in the moving sofa problem*. https://www.math.ucdavis.edu/~romik/data/uploads/papers/sofabounds.pdf

In 2018, the upper bounds of the maximum area that could go around a right-angled corner was reduced from 2.8284 to 2.37 by Dan Romik and Yoav Kallus. This value was closer to the existing discovered maximum area that could move around a right-angled corner.

Note: the width of the corridor is 1.

3. Methodology

The various mathematical concepts used are:
a) Geometry
b) Trigonometry
c) Calculus

3.1 Geometry

The table (Figure 2) summarises the formulae for coordinate geometry. All angles are in radians unless stated otherwise.

General Line Equation	$y = mx + c$ $where\ m\ is\ the\ slope\ and\ c\ is\ the\ y - intercept$
Slope Formula	$Slope\ m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$
Mid-point Formula	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance Formula	$\sqrt{\left(x - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

Figure 2 - Formulae for Coordinate Geometry

3.2 Trigonometry

The Law of cosines states that $a^2 = b^2 + c^2 - 2bc\cos(A)$ which denotes the angle contained between sides of lengths and opposite the side of length
If the lengths of all three sides of any triangle are known, then
$a = b\cos(C) \pm \sqrt{c^2 - b\sin^2(C)}$
These two formulas are extremely useful for our project. We can also derive the Pythagoras theorem from the first formula when A=90°.

We are also using the trigonometry rule $\tan(\theta) = \frac{opp}{adj}$

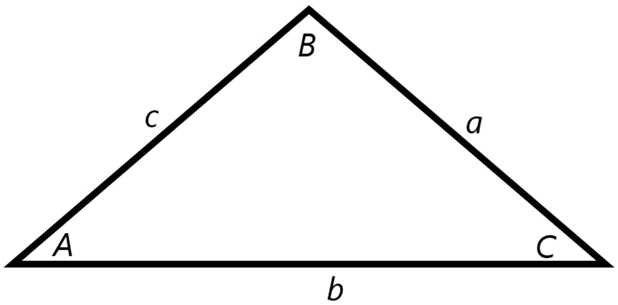


Figure 3

3.3 Calculus

We used differentiation to solve for the minimum length of a line going through point (a,b).
Below is a summary of Differentiation rules used

Power Rule	$if\ \ f(x) = x^n\ then\ f'(x) = nx^{n-1}$
Constant Rule	$if\ f(x) = c\ then\ f'(x) = 0$
Quotient Rule	$if\ \ h(x) = \frac{f(x)}{g(x)}\ then\ h'(x) = \frac{(f'(x)g(x) - f(x)g'(x))}{g(x)^2}$
Chain Rule	$if\ h(x) = f(g(x))\ then\ h'(x) = f'(g(x))g'(x)$

Figure 4

4. Findings and Results

4.1 Research Statement 1

To investigate the formula/e to move a 3-sided shape (n=3) around Right-Angle, Acute-angle and Obtuse-angle Corners

We only found consistent results with equilateral triangles

N = width of corridor

$A = B = C \text{ where } A,\ B \text{ and } C \text{ are sides of a triangle}$

For an equilateral triangle to go around a right-angled corridor, it needs to satisfy the following.

1. Suppose side A is the breadth of the triangle and H is the height with with respect to side A. Either A<N or H<N
2. $C < 2 \cdot \sqrt{2n^2}$

4.2 Research Statement 2

To investigate the formula/e to move a 4-sided shape (n=4) around Right-Angled, Acute-angled and Obtuse-angled Corners

This includes all 4-sided shapes with at least 1 pair of parallel lines and is not irregular.

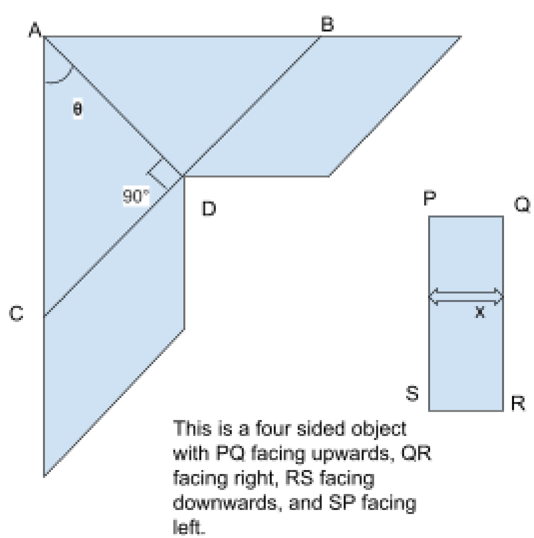


Figure 5

Assume the length of AD is known as the contractors should know it using the blueprints.
In order to optimise movement, the breath of the object should be parallel to line AB.
Furthermore, the longest line should be QR and facing right, assuming the corridor faces right.
If the shape can fit into triangle ABC, it can go through the corridor. Triangle ABC is a isosceles triangle such that $AB = AC$ and $CD = BD$, additionally, $\angle a = \angle d$.
If the corridor is a right angle, we can use Pythagoras theorem to calculate BC with $\sqrt{AB^2 + AC^2} = BC$.

If the corridor is acute or obtuse, BC can be calculated using trigonometry. Firstly, $\theta = \frac{\angle a}{2}$.
CD is the opposite and AD is the adjacent. We can use the function $\tan(\theta) = \frac{opp}{adj}$. Since we know the length of the adjacent, which is AD, we can take $\tan(\theta) \times adj = opp$ and $\tan(\theta) \times AD = CD$. Since $BC = CD + BD$ and $CD = BD$, to find BC, we can take $\tan(\theta) \times 2 \times AD = BC$.
Firstly, $\frac{PS + QR}{2} + PQ + SR \leq BC$.
Next, x must be less than the width of the corridor.
Lastly, the maximum area of the object is $\frac{1}{2} \times AD \times BC$.

4.2.2 Methodology and findings(square and rectangle with different width)

For a triangle to go around a right-angled corridor, it needs to satisfy the following.
For a rectangle to go around a right-angled corner with different widths we are proving the optimal length of a line to go around a right-angled corner.
Let's use the cartesian coordinate system and imagine a point (a,b), where the line has to pass through this point and the length is calculated. What is the optimal length of the line?

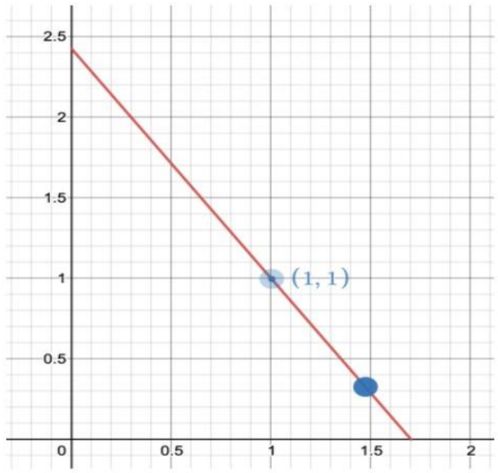


Figure 6

As seen from the diagram above, as this line pivots around a point (1,1), we try to find the dimensions of the minimum length of the red line.
Using the formula $y = mx + b$

$when\ x = 0,$

$y - a = m(0 - b)$

$y = a - bm$

$when\ y = 0$

$0 - b = m(a - 3)$

$x = \frac{(bm - a)}{m}$

$$c = \sqrt{\left(\frac{bm - a}{m}\right)^2 + (a - bm)^{2}}$$

$$c' = \frac{(bm - a)(bm^3 + a)}{(m^3\sqrt{\frac{bm - a^2}{m^2}(a - bm^2)}}$$

$when\ c' = 0,$

$$m = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Now we can use the width of the corridor on both sides and find the gradient . Now, we know the gradient m so we can now solve the problem.

Find l_1 *and* f_1

Greaterwidth refers to the width of the corridor with a greater width than the width of the other side of the corridor.

Smallerwidth refers to the width of the corridor with a smaller width than the width of the other side of the corridor.

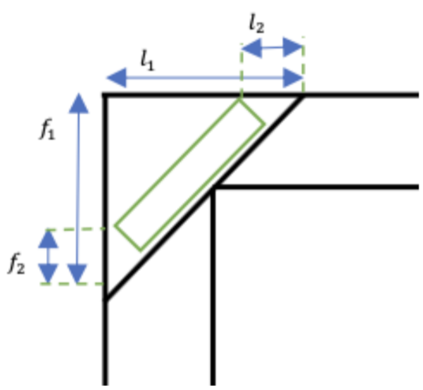


Figure 7

$$l_1 = greaterwidth \cdot m + smallerwidth$$

$$f_1 = \frac{smallerwidth}{m} + greaterwidth$$

let the width of the rectangle be w

$$l_2 = \sqrt{\left(\frac{w}{m}\right)^2 + w^2}$$

$$f_2 = \sqrt{(wm)^2 + w^2}$$

Now we find the maximum length of the rectangle

$$\sqrt{(f_1 - f_2)^2 + (l_1 - l_2)^2}$$

4.3 Research Statement 3

To investigate the formula/e to move a *n-sided* shape ($n = \infty$) around the right-angle, acute-angle and obtuse-angle corners.

For $n = \infty$, we considered elliptical and circular shapes.

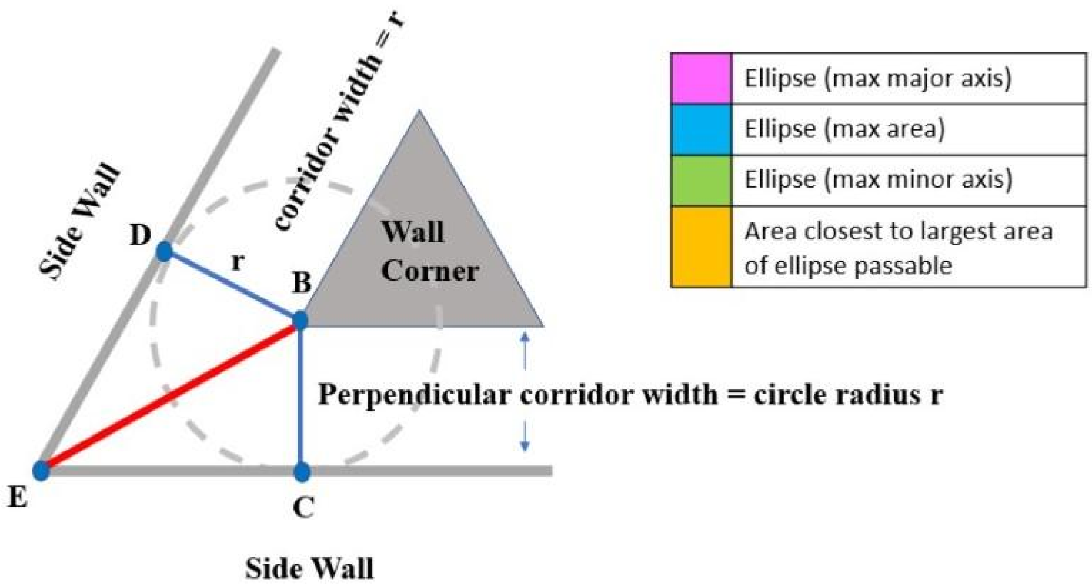


Figure 8 - Legend for Figures 9,11, 13 and 14

For corner angles, the width of shapes must be less than corridor width (r).

Referencing Figure 8, these shapes (in Figures 9, 11, 13 and 14) pivot/turn at corner B, where circle B with radius BC equal to corridor width r. To find the largest area of ellipse ($\text{Ellipse}_{\text{max}}$) and circle ($\text{Circle}_{\text{max}}$) passable, we find the maximum major axis ($r_{\text{major max}}$) and minor axis ($r_{\text{minor max}}$) and the maximum radius passable using geometry and Geogebra. We then compare the areas of the shapes and the turning regions to obtain our results.

4.3.1 Findings and Results

<u>Right-Angle Corner</u>

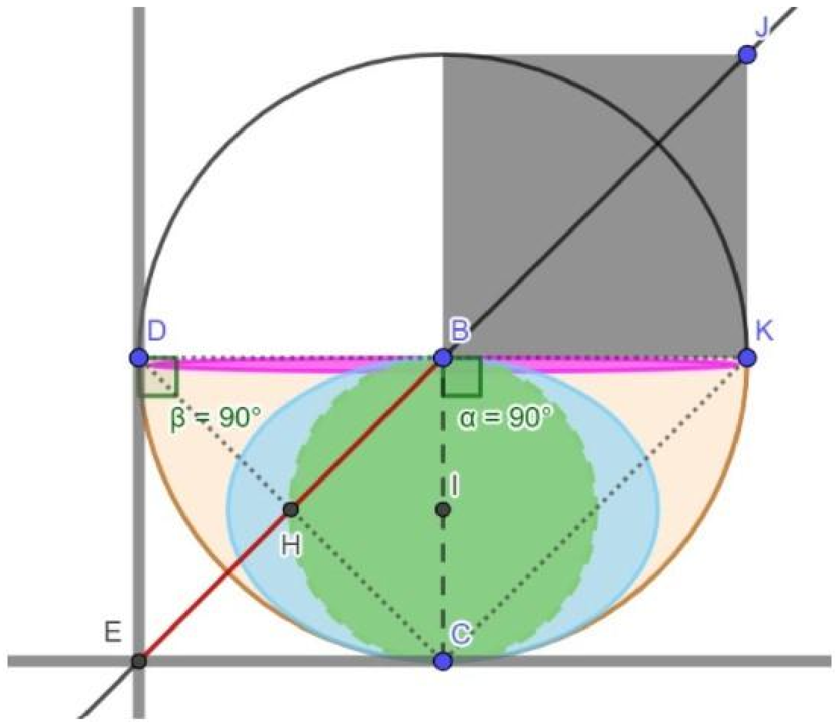


Figure 9- $n = \infty$ shapes passing the right-angle corner(1.57 as unable to convert 90° to radian in Geogebra Classic).

Referencing Figure 9,

$$r_{\text{major max}} = \frac{1}{2}DK = r \quad \text{and} \quad r_{\text{minor max}} = \frac{1}{2}BC = \frac{1}{2} \times r = \frac{r}{2}.$$

Since all angles subtended by DK on a semicircular arc are right-angle triangles on the same base, maximum area of triangle is $\frac{1}{2} \times DK \times BC = \frac{1}{2} \times 2r \times r = r^2$.

Hence, comparing areas of turning regions to shapes (Figure 10):

Area of Turning Region	$Square\ BCED = BC \times BD = r^2$ where $corridor\ width\ =\ radius\ r$
	$Semicircle\ B = \frac{1}{2} \times \pi \times r^2 = \frac{\pi r^2}{2}$ where radius = r
	$\triangle DKC = \frac{1}{2} \times DK \times BC = \frac{1}{2} \times 2r \times r\ =\ r^2$, where $r = corridor\ width$
Area of Shapes	$Circle_{max} = \pi r_{max}{}^2 = \frac{\pi r^2}{4}$ where radius = $\frac{r}{2}$
	$Ellipse_{max} = \pi r_{major} r_{minor}$ where major axis $< r$ and minor axis $\leq \frac{r}{2}$
If major axis is at maximum ($< r$)	$Ellipse_{major\ max} = \pi r_{major\ max} r_{minor}$
If minor axis is at maximum ($= \frac{r}{2}$)	$Ellipse_{minor\ max} = \pi r_{major} r_{minor\ max} = \pi r_{major} \frac{r}{2}$

Figure 10- Table of Areas of Turning Region (Right-Angle Corner) to Shapes

From Figure 10, we conclude that:

$$Area\ of\ Ellipse_{major\ max} < Circle_{max} < \triangle DKC \leq Square\ BCED < Ellipse_{minor\ max} \leq Ellipse_{max} < \ Semicircle\ B$$

Thus, $largest\ passable\ area(shape)\ <\ semicircle\ B$

(Refer to Annex 1 - Table 1)

Hence, for $n = \infty$ shapes to pass through right-angle corners:

1. Area of ellipse $\leq \frac{\pi r^2}{2}$,

   $where\ r = corridor\ width\ and\ maximum\ major\ axis\ < r\ while\ maximum\ minor\ axis\ \leq \frac{r}{2}$

2. $Area\ of\ circle \leq \frac{1}{2}\pi\left(\frac{r}{2}\right)^2\ or\ \pi\frac{r^2}{8}$, $where\ r \leq \frac{1}{2}\ corridor\ width$

Acute-Angle Corner

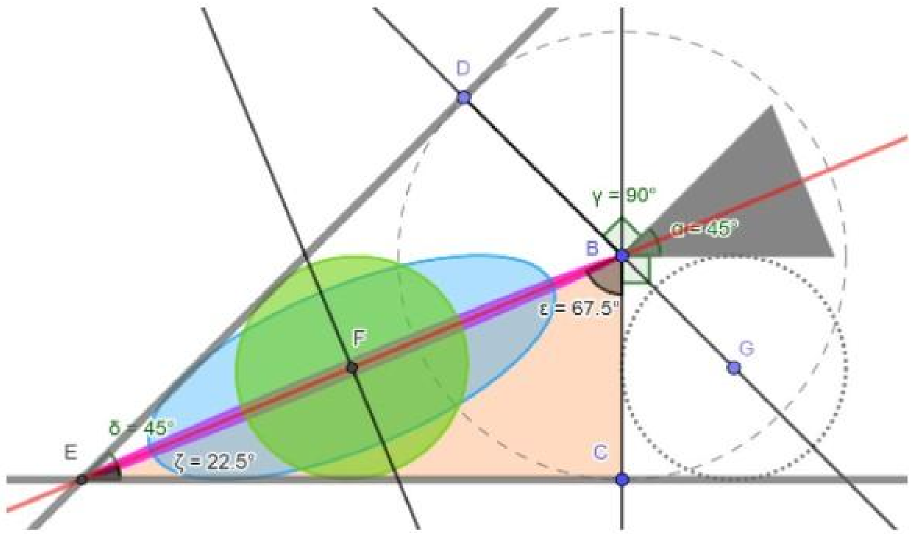


Figure 11- $n = \infty$ shapes passing the acute-angle corner(0.79 as unable to convert 45° to radian in Geogebra Classic)

Referencing Figure 11,

$$r_{major\ max} = \frac{1}{2}EB = EF \quad \text{and} \quad r_{minor\ max} = \frac{1}{2}BC = \frac{1}{2} \times r = \frac{r}{2}$$

Applying Sine Rule,

$$\frac{EC}{\sin(1.18)} = \frac{r}{\sin(0.39)} \quad where\ r = \ corridor\ width$$

$$EC = \frac{r}{\sin(0.39)} \times \sin(1.18)$$

Alternatively, using Pythagoras Theorem,

$$EC = \sqrt{(EB^2 - BC^2)} = \sqrt{(EB^2 - r^2)}$$

Hence, comparing areas of turning regions to shapes (Figure 12):

Area Of Turning Region	$\triangle EBC = \frac{1}{2} \times EC \times BC = \frac{1}{2} \times \left(\frac{r}{\sin(0.39)} \times \sin(1.18) \right) \times r = 1.207r^2$ OR $\triangle EBC = \frac{1}{2} \times \sqrt{(EB^2 - r^2)} \times r = \frac{1}{2} r \sqrt{(EB^2 - r^2)}$
	$Semicircle\ B = \frac{1}{2} \pi \times BC \times BC = \frac{1}{2} \pi r^2$ where BC = corridor width = r
	$Circle\ G = \pi \times \frac{1}{2} BC \times \frac{1}{2} BC = \pi \times \frac{r}{2} \times \frac{r}{2} = \frac{\pi r^2}{4}$
Area of Shapes	$Circle_{max} = \pi r_{max}{}^2 = \frac{\pi r^2}{4}$ where radius $= \frac{r}{2}$
	$Ellipse_{max} = \pi r_{major} r_{minor}$ where major axis $\leq \frac{EB}{2}$ and minor axis $\leq \frac{r}{2}$
If major axis is at maximum ($= \frac{EB}{2}$)	$Ellipse_{major\ max} = \pi r_{major\ max} r_{minor} = \pi \frac{EB}{2} r_{minor}$
If minor axis is at maximum ($= \frac{r}{2}$)	$Ellipse_{minor\ max} = \pi r_{major} r_{minor\ max} = \pi r_{major} \frac{r}{2}$

Figure 12- Table of Areas of Turning Region (Acute Corner) to Shapes

From Figure 12, we conclude that:

$$Area\ of\ Ellipse_{major\ max} < Circle_{max} \leq Circle\ G \leq Ellipse_{minor\ max} < Ellipse_{max} < \triangle EBC < Semicircle\ B$$

Thus, $largest\ passable\ (shape)\ area < \triangle EBC$

(Refer to Annex 1- Table 2)

Hence, for $n = \infty$ shapes to pass through acute-angle corner,

1. $Area\ of\ ellipse \leq \triangle EBC$, where the maximum major axis $\leq \frac{1}{2} EB$ while the maximum minor axis $\leq \frac{1}{2} BC\ or\ \frac{r}{2}$.
2. $Area\ of\ circle \leq \triangle EBC$

Obtuse- Angle Corner

Figures 13 (2.36) and 14 (2.76) show that turning movement can occur within rhombus *BGEF* where EB bisects $\angle FEG$, forming 2 equal isosceles Δs FEB and GEB.

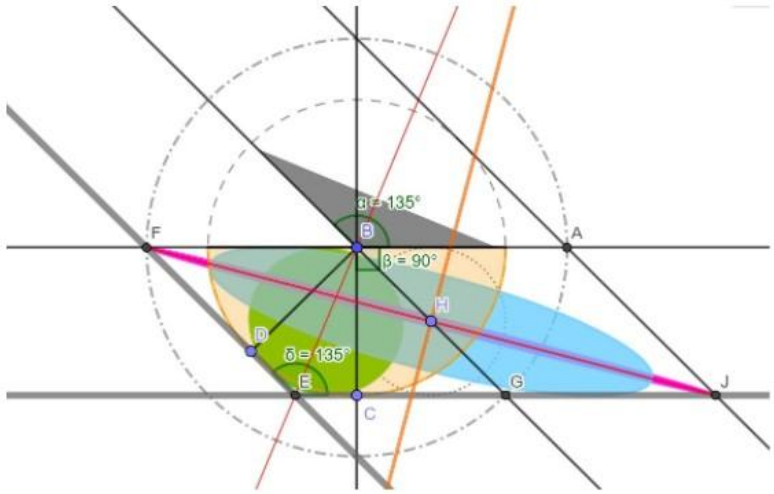


Figure 13- $n = \infty$ shapes passing the obtuse-angle corner (2.36 as unable to convert 135° to radian in Geogebra Classic)

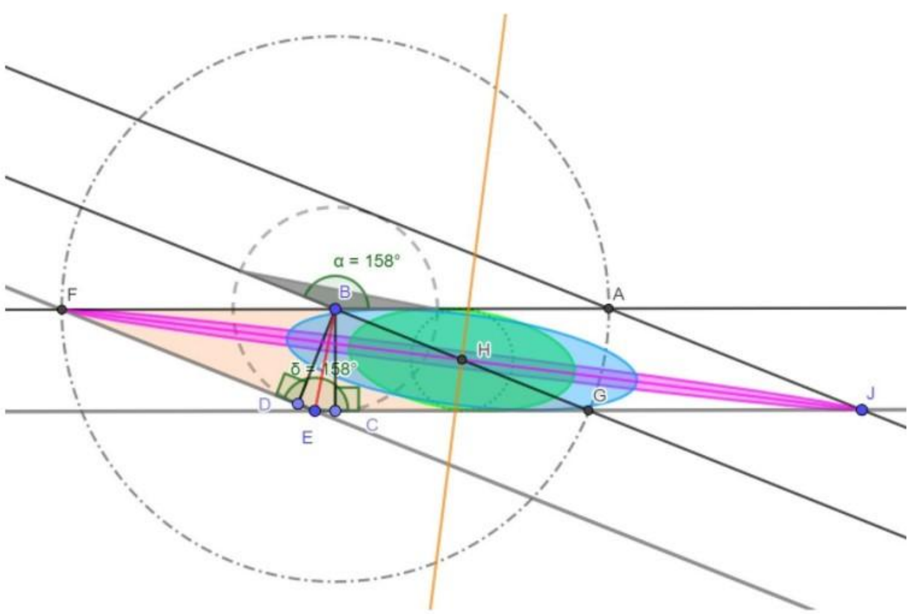


Figure 14--$n = \infty$ shapes passing the obtuse-angle corner (2.76 as unable to convert 158° to radian in Geogebra Classic)

Instead of $\frac{1}{2}EB$, $r_{major\ max} = \frac{1}{2}FJ$ and $r_{minor\ max} = \frac{1}{2}BC = \frac{1}{2} \times r = \frac{r}{2}$

Hence, comparing areas of turning regions to shapes (Figure 15):

Area of Turning Region	$Rhombus\ BGEF = BC \times BG$
	Semicircle B $= \frac{1}{2}\pi \times BC \times BC = \frac{1}{2}\pi r^2$ $where\ BC = corridor\ width = r$
	Circle H $= \pi \times \frac{1}{2}BC \times \frac{1}{2}BC = \pi \times \frac{r}{2} \times \frac{r}{2} = \frac{\pi r^2}{4}$
Area of Shapes	$\text{Circle}_{\text{max}} = \pi r_{max}{}^2 = \frac{\pi r^2}{4}$ where radius $= \frac{r}{2}$
	$\text{Ellipse}_{\text{max}} = \pi r_{major} r_{minor}$ where major axis $\le \frac{FJ}{2}$ and minor axis $\le \frac{r}{2}$
If major axis is at maximum ($= \frac{FJ}{2}$)	$\text{Ellipse}_{\text{major max}} = \pi r_{major\ max} r_{minor} = \pi \frac{FJ}{2} r_{minor}$
If minor axis is at maximum ($= \frac{r}{2}$)	$\text{Ellipse}_{\text{minor max}} = \pi r_{major} r_{minor\ max} = \pi r_{major} \frac{r}{2}$

Figure 15- Table of Areas of Turning Region (Obtuse Corner) to Shapes

Referencing Figures 13 and 14, the largest turning regions are different because as the obtuse angle increases, FJ and the area of rhombus BGEF also increases, but the area of semicircle B remains constant since radius remains unchanged:

$$Ellipse_{max} = \pi r_{\text{major}} r_{\text{minor}} \text{ where major } axis \le \frac{FJ}{2} and\ minor\ axis \le \frac{r}{2}$$

Using Cosine Rule,

$FJ^2 = FE^2 + EJ^2 - 2(FE)(EJ)\cos(FEG)$

$\boldsymbol{Referencing\ Figure\ 13\ (2.36)},$

$$cos(CBG) = cos(0.79) = \frac{BC}{BG} = \frac{r}{BG}$$

thus, $BG = \dfrac{r}{\cos(0.79)} = 1.42r \quad where\ r\ =\ corridor\ width$

hence, $\boldsymbol{Rhombus\ BGEF = BC \times BG = r \times 1.42r = 1.42r^2}$

$$FJ\ =\ \sqrt{\left(BG^2+(2BG)^2-2(BG)(2BG)cos(2.36)\right)}\ =\ 1.96r \text{ where } BG = FE\ and\ 2BG\ =\ EJ$$

Thus, $r_{major} \le \dfrac{FJ}{2} = 1.96r/2\ =\ 0.98r$

Hence, $Ellipse_{max}\ =\ \pi(0.98r)(r_{minor})$ where $r_{minor} \le \dfrac{r}{2}$

From Figure 13, we conclude that for obtuse ≤ 2.36:

$$Area\ of\ Ellipse_{major\ max} < Circle_{max} \le Circle\ H \le Ellipse_{minor\ max} < Rhombus\ BGEF < Ellipse_{max} <\ Semicircle\ B$$

Thus, $largest\ passable(shape)\ area <\ semicircle\ B$

(Refer to Annex 1-Table 3)

$\boldsymbol{However,\ referencing\ Figure\ 14\ (2.76)},$

$$cos(CBG) = cos(1.19) = \frac{BC}{BG} = \frac{r}{BG}$$

thus, $BG = \dfrac{r}{\cos(1.19)} = 2.69r \quad where\ r\ =\ corridor\ width$

hence, $\boldsymbol{Rhombus\ BGEF = BC \times BG = r \times 2.69r = 2.69r^2}$

$$FJ\ =\ \sqrt{\left(BG^2+(2BG)^2-2(BG)(2BG)cos(2.76)\right)}\ =\ 7.94r\ where\ BG = FE\ and\ 2BG = EJ$$

In this case, $r_{major} \le \dfrac{FJ}{2}\ =\ \dfrac{7.94r}{2}\ =\ 3.97r$

Hence, $Ellipse_{max}\ =\ \pi(3.97r)(r_{minor})\ where\ r_{minor} \le \dfrac{r}{2}$

From Figure 14, we conclude that for obtuse >2.36:

$$Area\ of\ Ellipse_{major\ max} < Circle_{max} \le Circle\ H \le Ellipse_{minor\ max} < Semicircle\ B < Ellipse_{max} < Rhombus\ BGEF$$

Thus, largest passable(shape) area < rhombus $BGEF$

(Refer to Annex 1-Table 4)

Hence, for $n = \infty$ shapes to pass through obtuse-angle corner,

1. $For\ Obtuse - Corners\ (\le 2.36),\ area\ of\ ellipse \le \frac{1}{2}\pi r^2$,

   $where\ r = corridor\ width\ and\ the\ maximum\ major\ axis \le \frac{FJ}{2}$

   $while\ the\ maximum\ minor\ axis \le \frac{r}{2}$.

2. $For\ Obtuse - Corners > 2.36,\ the\ area\ of\ ellipse \le rhombus\ BGEF\ (or\ BC \times BG)$

   $where\ BC = corridor\ width\ (r)\ and\ the\ maximum\ major\ axis \le \frac{FJ}{2}$

   $while\ the\ maximum\ minor\ axis \le \frac{r}{2}$

3. For all obtuse corners, $area\ of\ circle \le \frac{1}{2}\pi(\frac{r}{2})^2\ or\ \pi\frac{r^2}{8}\ where\ r \le \frac{1}{2} corridor\ width$

4.4 Research Statement 4

To investigate the formula/e to move *irregular* shapes around a $\frac{\pi}{2}$ corridor.

We define the sofa to be a closed, bounded region in a plane whose boundary is a simple closed curve. The hallway is defined with the following:

$$A = (r\cos\alpha, t\sin\alpha)$$
$$A' = \left(r\cos\alpha + \sqrt{2}\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right), t\sin\alpha + \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right)$$
$$B = \left(r\cos\alpha - \frac{t\sin\alpha}{\tan\left(\frac{\alpha}{2}\right)}, 0\right)$$
$$B' = \left(r\cos\alpha - \frac{t\sin\alpha}{\tan\left(\frac{\alpha}{2}\right)} - \frac{1}{\sin\left(\frac{\alpha}{2}\right)}, 0\right)$$
$$C = \left(r\cos\alpha + t\sin\alpha\tan\left(\frac{\alpha}{2}\right), 0\right)$$
$$C' = \left(r\cos\alpha + t\sin\alpha\tan\left(\frac{\alpha}{2}\right) + \frac{1}{\cos\left(\frac{\alpha}{2}\right)}, 0\right)$$

We can move the hallway instead of the sofa to obtain shapes that can pass through the hallway. The customer inputs a shape it will check for a bigger or same shape that can pass through and if it finds it, it will output,“Your sofa can pass through”, else, it will output,“Your sofa cannot pass through”. Figure 16 will illustrate the findings

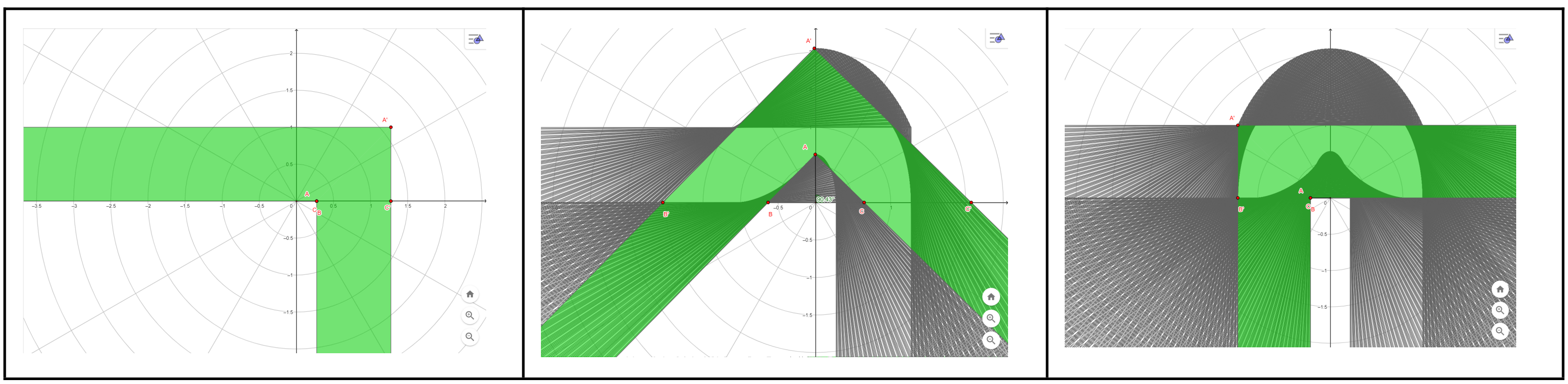


Figure 16

From that we can see that there are two functions p(controls the x axis) and q(controls the y axis) and let's consider them to be p= f(α) and p= g(α) and α is the ∠AOC and it is also ∠ADC where D changes location on x axis for r≠t and the constraints are 0≤r ≤3 and 0≤t≤2. When r=t=0, we get a semicircle as there is only rotation but no movement forward. The formula for moving the hallway is:

$S \subseteq \bigcap\limits_{0 \le \theta \le \pi/2} x(\theta) + R_\theta(H)$ where x(θ) is the rotation path where $x : [0, \pi/2] \to \mathbb{R}^2$ and $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$

is a clockwise rotation about the origin by an angle of θ and $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$.

4.5 Limitation and Further Extension

Our project is limited by the 2-dimensional *n-sided* shapes and same-width corridors on stepless surfaces (to keep turning angles constant), which may not be realistic representations.

Instead of a mix of *n-sided* regular and irregular polygons, our project can be further extended to cover *n-sided* regular polygons and *n-sided* irregular polygons.

4.6 Animation - Geogebra

Animation helps in our problem visualisation and application of findings and it helps the viewers understand our formulas better.

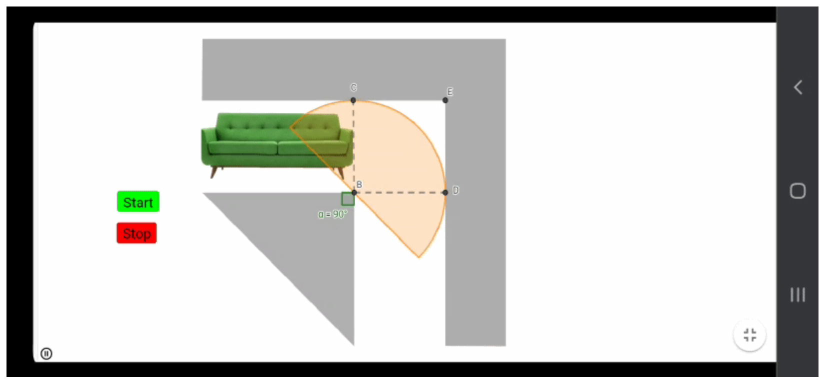
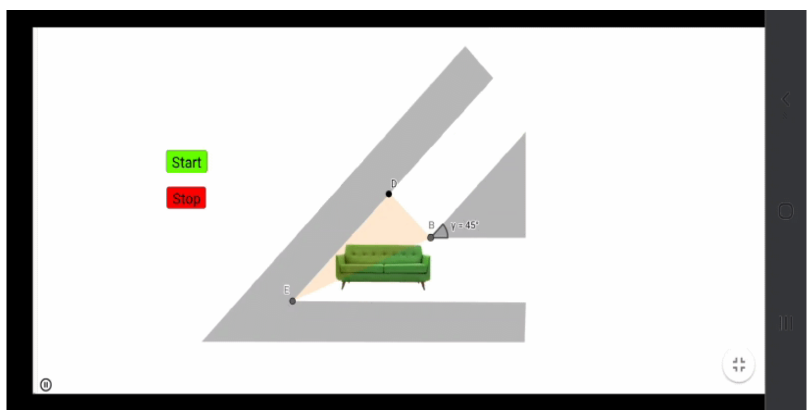
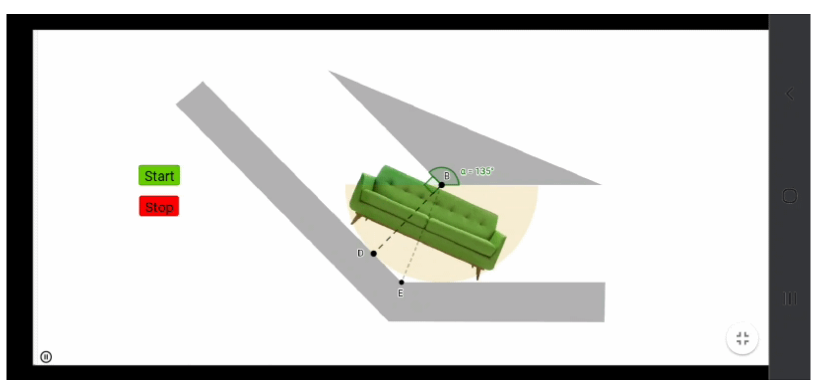
Right-Angle Corner (1.57 *or* 90°)	
Acute-Angle Corner (0.79 *or* 45°)	
Obtuse-Angle Corner (2.36 *or* 135°)	

Figure 17

4.7 Simulation

Python

We wrote a Python Program to calculate the dimension of a square or rectangle moving around a right-angled corner. Here is the example of our program:

```
Type in the longer width of the corridor: 3
Type in the shorter width of the corridor: 3
what is the width of the square: 1
the maximum length is 6.485281374238S7
```

Figure 18

5. Timeline

Week	Tasks (With Mentor's Guidance)
March Holidays	Explore Possible Math Topics Proposal Slides
T2W2 - T2W3	Complete Proposal Evaluation Slides Rehearsals Proposal Evaluation Presentation Research
T2W5 - June Holidays	More Research
June Holidays- T3W6	More Research Final-Evaluation Slides Written Report
T3 W7	Complete Final Evaluation Slides/Written Report
T3W7 - T3W8	Rehearsals
T3W7	Written Report Submission
T3W8	Final Evaluation Presentation

Figure 19--Timeline

6. Conclusion

Here is a summary of all the research statement findings.

Research statement number	Results
1	Suppose side A is the breadth of the triangle and H is the height with with respect to side A. Either A<N or H<N $C<2\cdot\sqrt{2n^2}$
2	A set of conditions were found for the shape to be able to pass through, which were, (referring to the diagram above), $\frac{PS+QR}{2}+PQ+SR\leq BC$, and the maximum area was $\frac{1}{2}\times AD\times BC$, lastly the x had to be less than the width of the corridor.
3	For $n=\infty$ (elliptical shapes) - right-angle : $largest\ passable\ area(shape) < semicircle\ B$ - acute - angle: $largest\ passable\ (shape)\ area < \Delta EBC$ - Obtuse- angle ≤ 2.36: $largest\ passable(shape)\ area < semicircle\ B$ - Obtuse- angle >2.36: $largest\ passable(shape)\ area < rhombus\ BGEF$ For $n=\infty$ (circular shapes) - right -angle: $largest\ passable(shape)\ area \leq \frac{1}{2}\pi\left(\frac{r}{2}\right)^2$ or $\pi\frac{r^2}{8}$ where $r\leq\frac{1}{2}$ corridor width - acute -angle: $largest\ passable\ (shape)\ area \leq \Delta EBC$ - obtuse-angle: $largest\ passable\ (shape)\ area \leq \frac{1}{2}\pi(\frac{r}{2})^2$ or $\pi\frac{r^2}{8}$ where $r\leq\frac{1}{2}$corridor width
4	Instead of moving the sofa, we move the hallway. We will run a computer simulation check for a bigger or the same shape that can pass through and if it finds it, it will output,"Your sofa can pass through", else, it will output,"Your sofa cannot pass through". The constraints are that $0\leq r\leq 3$ and $0\leq t\leq 2$.
Extra (different widths)	Maximum possible length of the rectangle at width w= $\sqrt{(f_1-f_2)^2+(l_1-l_2)^2}$

We could not find a trend in all the results. Although we could not find a general trend, the results of this project can still be useful as in our results we separately accounted for three and four shaped furniture, and also circular objects , hence our results can still be applied to real life situations.

We have managed to make a formula for moving squares and rectangles around right-angled corners, which is very applicable to real life situations. For example, when moving house, objects are placed in boxes. We can find the maximum dimensions of these boxes.

There is some possibility of a project extension such as the ambidextrous sofa but it is unlikely to take place.

7. Annex

Annex 1 -- Geogebra Geometry Measurements of Ellipse

Legend (for all tables)

- Ellipse (max major axis)
- Ellipse (max area)
- Ellipse (max minor axis)
- Area closest to largest area of ellipse passable

Annex 1 - Table 1 : Ellipse Measurements for Right-Angle Corner (1.57 or 90 °)

Dimensions (unit)			**Area (unit²)**		
Corridor width (r)	3		Square BCED		9
Hypotenuse EB	4.24		Δ CDK		9
			Semicircle B (r=3)		14.14
			Circle I (r=1.5)		7.07
Measurements of Ellipse					
Foci	2.9	2.8	2.7	2.5	1.5
Major axis	2.9	2.89	2.77	2.58	2.12
Minor axis	0.07	0.75	0.62	0.62	1.5
Area of ellipse	0.63	7.01	5.42	5.03	10

Legend (for all tables)

- Ellipse (max major axis)
- Ellipse (max area)
- Ellipse (max minor axis)
- Area closest to largest area of ellipse passable

Annex 1 - Table 2 : Ellipse Measurements for Acute-Angle Corner (0.79 or 45 °)

Dimensions (unit)			**Area (unit²)**							
Corridor width (r)	3		Δ BCE		10.86					
Hypotenuse EB	7.84		Semicircle B (r=3)		14.14					
			Circle G (r=1.5)		7.07					
Measurements of Ellipse										
Foci	3.90	3.50	3.00	2.90	2.70	2.50	2.00	1.50	1.00	0.50
Major axis	3.92	3.56	3.15	3.07	2.91	2.75	2.38	2.04	1.76	1.56
Minor axis	0.10	0.65	0.96	1.02	1.09	1.15	1.28	1.38	1.46	1.48
Area of ellipse	1.25	7.35	9.39	9.75	9.96	9.94	9.61	8.85	8.03	7.26

Legend (for all tables)

Pink	Ellipse (max major axis)
Blue	Ellipse (max area)
Green	Ellipse (max minor axis)
Orange	Area closest to largest area of ellipse passable

Annex 1 : Table 3 - Ellipse Measurements for Obtuse-Angle Corner (2.36 or 135°)

Dimensions (unit)		Area (unit²)	
Corridor width (r)	3	Rhombus BFEG	12.73
Hypotenuse EB	3.25	Isoceles Δ BEG = BEF	6.36
Side BF=BG (rhombus BFEG)	4.24	Right-angle Δ ECB	1.86
Diagonal FG (rhombus BFEG)	7.84	Parallelogam FAJE	25.45
Side EC (Δ ECB)	1.24	Δ FJE (parallelogram)	12.73
Diagonal FJ (parallelogram)	**11.87**	Semicircle B (R=4.24)	28.25
Diagonal EA (parallelogram)	6.25	Semicircle B (r=3)	14.14
Side FA (parallelogram)	8.48	Circle H (r=1.5)	7.07

Measurements of Ellipse	Rhombus BFEG											Parallelogramgram FAJE				
Foci	3.90	3.50	3.00	2.70	2.50	2.40	2.20	2.00	1.50	1.00	0.50	5.90	5.50	5.00	4.80	4.50
Major axis	**3.92**	3.57	3.15	2.90	2.75	2.68	2.52	2.38	2.04	1.75	1.56	5.90	5.52	5.06	4.89	4.60
Minor axis	0.15	0.69	0.98	1.07	1.14	1.20	1.23	1.28	1.39	1.45	1.47	0.11	0.52	0.80	0.91	0.97
Area of ellipse	1.90	7.71	9.65	9.70	9.88	**10.07**	9.80	9.60	8.84	7.93	7.22	1.99	9.11	12.57	13.97	14.00

Legend (for all tables)

(pink)	Ellipse (max major axis)
(blue)	Ellipse (max area)
(green)	Ellipse (max minor axis)
(orange)	Area closest to largest area of ellipse passable

Annex 1 : Table 4 - Ellipse Measurements for Obtuse-Angle Corner (2.76 or 158 °)

Dimensions (unit)		**Area (unit²)**	
Corridor width (r)	3	Rhombus BFEG	24.07
Hypotenuse EB	3.06	Isoceles Δ BEG = BEF	12.04
Side BF=BG (rhombus BFEG)	8.03	Right-angle Δ ECB	0.90
Diagonal FG (rhombus BFEG)	15.74	Paralleloram FAJE	48.15
Side EC (Δ ECB)	0.6	Δ FJE (parallelogram)	24.07
Diagonal FJ (parallelogram)	**23.67**	Semicircle B (R=8.03)	101.33
Diagonal EA (parallelogram)	9.14	Semicircle B (r=3)	14.14
Side FA (parallelogram)	16.05	Circle H (r=1.5)	7.07

Measurements of Ellipse	**Rhombus BFEG**				**Parallelogramgram FAJE**								
Foci	4.00	3.00	2.00	1.00	2.90	3.00	3.20	4.00	5.00	6.00	7.00	8.00	11.83
Major axis	4.20	3.29	2.46	1.73	3.24	3.33	3.51	4.23	5.18	6.07	7.06	8.03	11.60
Minor axis	1.28	1.35	1.42	1.40	1.43	1.45	1.44	1.37	1.34	0.93	0.93	0.73	0.25
Area of ellipse	16.86	13.89	11.01	7.66	14.57	15.23	15.59	18.26	21.85	17.77	20.58	18.41	9.55

8. Acknowledgements and Credits

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