The Multidimensional Mystery

By: Group 8-29
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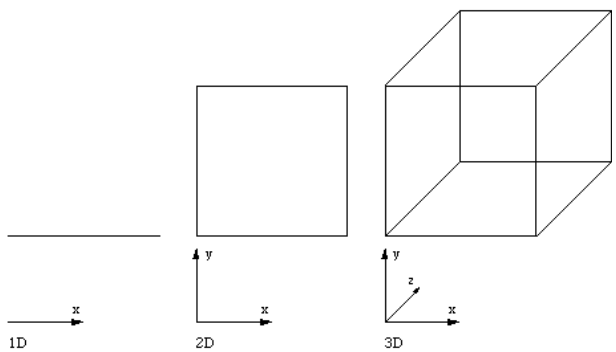
1. Introduction and Rationale

What is dimension?

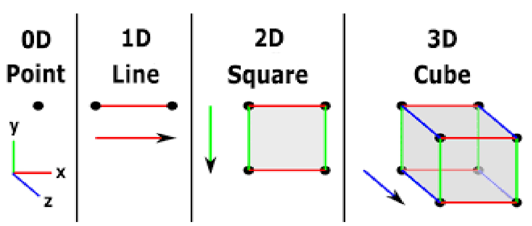
It refers to the least number of coordinates to specify any point within it.
For instance, a person in a 1D universe would only be able to move forward and backwards as the 1D universe is based on a line.
Following this principle, a person in a 2D universe would only be able to move forwards, backwards, to the left and to the right as a 2D universe is based on a flat plane without any height. Hence, people in 2D universes would not be able to move upwards or downwards.
Similarly, a person in a 3D universe would be able to move upwards, downwards, backwards, forwards, to the left and to the right as every location in a 3D universe would have 3 coordinates, which are along the x, y and z axes.



How many dimensions are there?

There are 10 dimensions according to the String Theory.

The 0D universe is a point.
The 1D universe is a line. (length)
The 2D universe is a plane. (area)
The 3D universe is space. (volume)



The 4D universe is time.
According to superstring theory, 5D and 6D are where the notions of possible worlds arise.

In the 6D universe, we could see planes of possible worlds.

In theory, if 5D and 6D are mastered, you can travel back in time or go to the future.

In the 7D universe, we have access to possible worlds starting with initial conditions and everything is different from the beginning of time.

In the 8D universe, we have a plane of such possible universe histories, each of which begins with different initial conditions and branches out infinitely.

In a 9D universe, we can compare all the possible universe histories, starting with all the different possible laws of physics and initial conditions.

In a 10D universe, we would arrive at the point in which everything possible and imaginable is covered. Beyond this, nothing else can be imagined by us, which makes it the natural limitation of what we can conceive in terms of dimensions.

Hence, are there any dimensions higher than 10?

According to different theories on dimensions by ancient physicists from centuries ago, the number of dimensions may range from 10 to even 26.

However, recently, physicists have determined that there are only a maximum of 11 dimensions as any dimension above 11 would be unstable and eventually collapse to the 11th dimension.

How did dimensions come about?

The modern concept of dimension started in 1863 with Maxwell, who synthesised earlier formulations by Fourier, Weber and Gauss. In doing so he added a nuance that we acknowledge today---whenever we refer to the dimensions of, say, g (≈ 9.81 m s^{-2}) as distance over time squared, rather than just the dimensional exponents (1, −2). By referring to the dimensions of a quantity, Maxwell seemed to imply that real things have natural dimensions.

What are the purposes of higher dimensions?

Based on Einstein‘s theory of special relativity, he thinks that the fourth dimension is time. Understanding these higher dimensions is of importance to mathematicians and physicists because it helps them understand the world.

2. Objectives

Research on questions we have about higher dimensions

1D: length

2D: length x breadth

3D: length x breadth x height

3D+1D:length x width x height x time

3D+2D: length x width x height x time x timeline

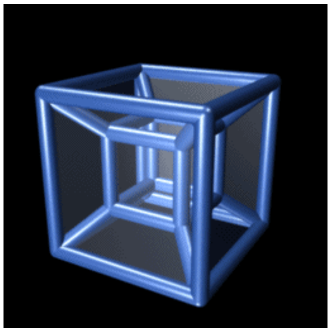
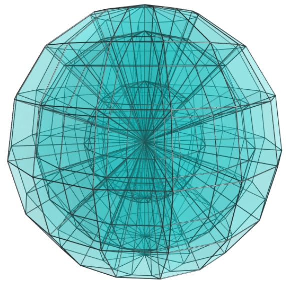
4D+2D: length x width x height x time x timeline x universe

5D+2D: length x width x height x time x timeline x universe x multiverse

How do dimensions apply to our daily lives?

Everything in our lives requires dimensions, namely the third dimension as we are dealing with shapes in this world. Everything originates from 3D shapes, from trees to houses, etc.

3. Literature Review

String theory

In physics, string theory is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings. String theory describes how these strings propagate through space and interact with each other.

String theory is a theory proposed by researchers as a way to describe the nature of the universe. It describes the different particles as vibrations of the string. Unfortunately, they require ten dimensions for all the math to work out, but the math in string theory does work out

String theory proposes that the fundamental constituents of the universe are one-dimensional “strings” rather than point-like particles. ... String theory also requires six or seven extra dimensions of space, and it contains ways of relating large extra dimensions to small ones

the names of the five string theories are: Type 1, Type IIA, Type IIB, SO(32) heterotic, and E8xE8 heterotic

In theoretical physics, type I string theory is one of five consistent supersymmetric string theories in ten dimensions. It is the only one whose strings are unoriented (both orientations of a string are equivalent) and which contains not only closed strings, but also open strings.

At low energies, type IIA string theory is described by type IIA supergravity in ten dimensions which is a non-chiral theory

The mathematical treatment of type IIA string theory belongs to symplectic topology and algebraic geometry, particularly Gromov–Witten invariants.

At low energies, type IIB string theory is described by type IIB supergravity in ten dimensions which is a chiral theory

At low energies, type IIB string theory is described by type IIB supergravity in ten dimensions which is a chiral theory

In string theory, a heterotic string is a closed string (or loop) which is a hybrid ('heterotic') of a superstring and a bosonic string. There are two kinds of heterotic string, the heterotic SO(32) and the heterotic E8 × E8, abbreviated to HO and HE.

String theory is a theory proposed by researchers as a way to describe the nature of the universe. It describes the different particles as vibrations of the string. Unfortunately, they require ten dimensions for all the math to work out, but the math in string theory does work out.

Based on a reputable youtube channel, minutephysics, there are three proposed theories of parallel universes.

1. Bubble universes: Our universe, along with others, are in a bubble or a black hole, so we cannot access another universe.
2. Extra Dimensions: Our universe is just one of many three dimensional universes, but there are another 6 dimensions that contain many universes. So if this is how the universe works, then in theory, we can travel to the other universes if we can go through these multiverse dimensions. String Theory gave rise to this idea.
3. Many worlds: In quantum mechanics, an atom is usually in a superposition, and universes branch off from there as things are entangled. In the famous Schrodinger’s cat thought experiment, the

atom is in a superposition of being both decayed and not decayed, the radiation detector and the cat is entangled with the atom, as if it decayed, the radiation will cause the detector to go off and the cat will die. If it didn't, none of those things would happen. We are also entangled with the cat as we are also made up of atoms. Thus, there are two copies, one with the cat dead, and one with the cat alive.

However, none of these theories have any proof, so all of them can be both real or fake. The universe can also be combinations of these three.

4. Intended Methodology

Our group had online zoom meetings for 1 hour every Monday starting from 1st March to discuss our project .Group leader will assign work for us to do. We also planned for meet-ups with our mentor every few weeks for her to review our progress. We created a google sheet to collate our contributions to the project.

Xing Cheng	Mu Yuan	Kai Shen	Khai Siang
Added on to all research questions	Provided the project idea	Research question 3	Research question 2
Created slides for evaluation (however info provided by whole group)	Provided the project title	Researched on polytopes	Researched about hypersphere
Looked up on how many dimensions there were	Researched on multiverses	Researched on the string theory	Researched about hypercube
String theory	Research question 1	Researched on 1st to 10th dimensions	Researched about Euclidean space
Searched for math equations for hypersphere	Researched on string theory	Researched on spatial and temporal dimensions	Researched about formulas for objects in other dimensions
Wrote the final edition of written report	Researched on lower dimensional viewpoints in higher dimensional space	Formula of 4D and 5D	Researched about how can we perceive objects in other dimensions
Researched on 4D	Researched on 3D objects	Number of points in each dimension (2^n point, nth dimension)	Researched on number of dimensions in project work evaluation
Researched on 5D	Researched on 4D objects	Edited the slides	Corrected and improved on the areas our project mentor pointed out
Researched on 6D	Researched on 5D objects		
Researched on 7D	Researched on 6D objects		
Researched on 8D	Researched on 7D objects		
	Researched on 8D objects		
	Researched on 9D objects		
	Researched on 10D		

	objects		
	Researched on dihedral angles		
	Researched on the possibility of higher dimensions		
	Researched on the size of dimensions		

5. Realistic Time Frame for Completion of Project

1st week of April: Project Evaluation

2nd week of April to 4th week of April: Research on examples of higher dimensions (For example: polytopes and klein bottle)

Entire month of May: Research on how dimensions are related to maths

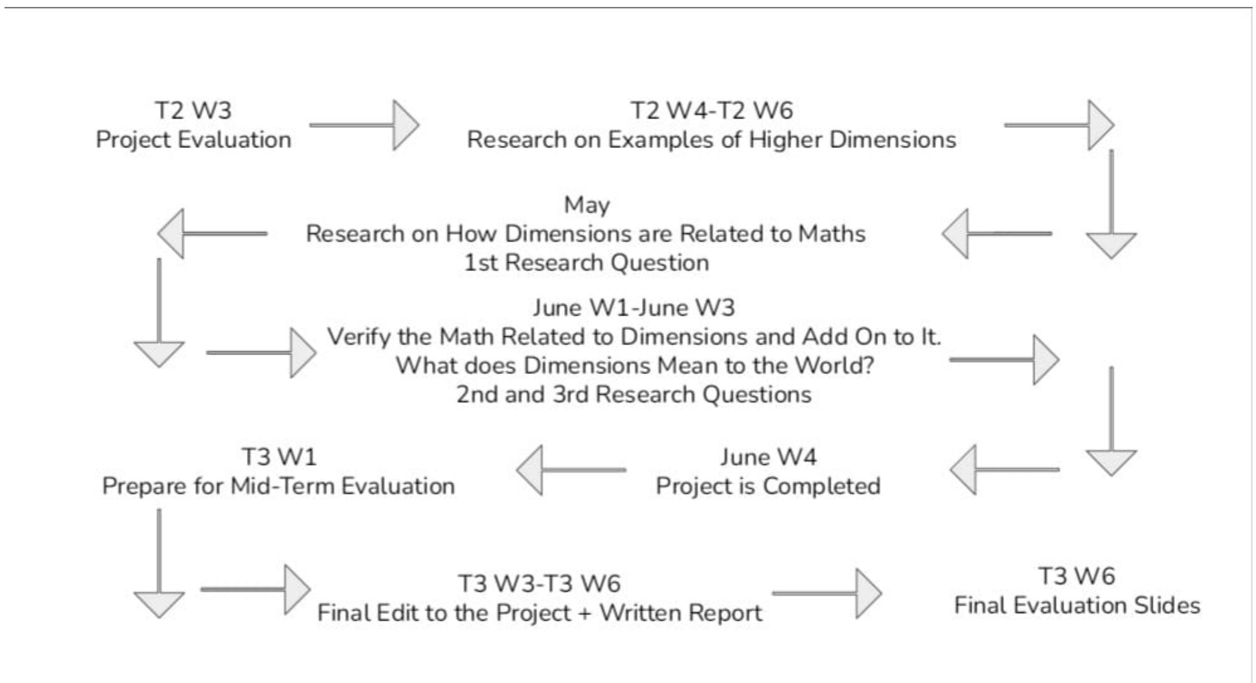
1st week of June to 3rd week of June: Verify the math related to dimensions and add on to it. What does dimensions mean to the world?

4th week of June: Project is completed

1st week of July: Prepare for Mid-Term Evaluation

3rd week of July to 1st week of August: Final edit to the project + Summary of Project

1st week of August: Final Evaluation Slides



We followed our timeline closely and were able to complete our project in time.

6. What are examples of objects in higher dimensions?

Polytopes

A polytope is the convex hull of finitely many points in a Euclidean space. The nts a, b ∈ Y, every point on the straight-line segment joining them is also in Y. The convex hull of a set of points X in Euclidean space is the smallest convex set containing X. Colloquially speaking, one way to define a polytope is as a finite set of points which have been shrink wrapped.

Examples:

Polygons, cubes, tetrahedrons, pyramids, and hypercubes, also known as tesseracts.

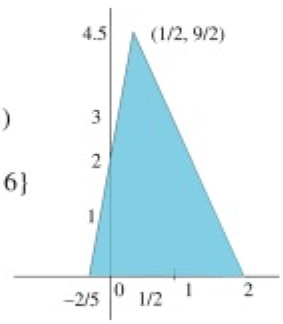
Another geometric definition of a polytope utilizes half-spaces, which are given by linear inequalities. If we take finitely many linear equalities such that the set of points which obey all of them is bounded, that set of points is a polytope.

Example:

Any polytope given by a convex hull can also be given in this manner

$$Conv(\{(-2/5, 0), (2, 0), (1/2, 9/2)\})$$

$$= \{(x, y) \mid y \geq 0 \;;\; y \leq 5x + 2 \;;\; y \leq -3x + 6\}$$

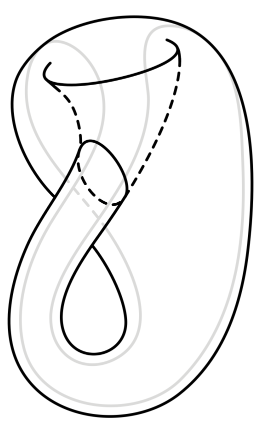


A polytope can also be described combinatorically as a partially ordered set, or more specifically a lattice. Each polytope is made up of smaller polytopes—its faces—ordered by containment.

For example, all of the corners and edges of an octagon are faces of the octagon.

The dimension of a polytope is the dimension of the smallest Euclidean space which could contain it. For example, the dimension of a pentagon is 2.

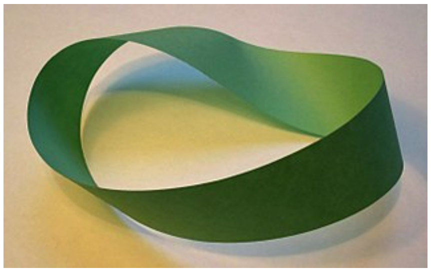
Klein bottle



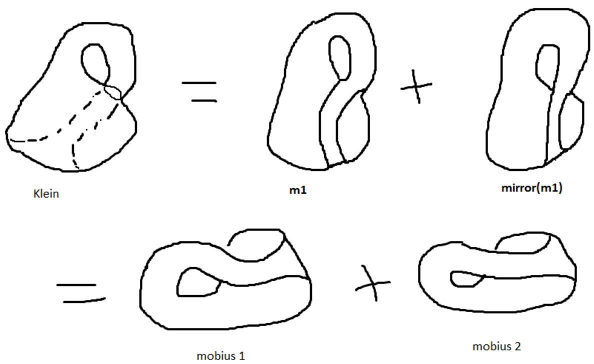
The Klein bottle does not intersect with itself in 4D (as shown by the darker outline), however 3D replicas (as shown by the lighter outline) need to have at least one self-intersection.

The Klein bottle is related to the Mobius Strip.

Mobius Strip



It is essentially made by twisting a piece of paper and connecting it.
The Klein Bottle is made up of two Mobius Strips



This is a simple drawing of how the Klein Bottle can be broken down into two Mobius Strips

This might be a better illustration of this concept of putting two Mobius Strips together to form one Klein Bottle.

7. Research questions

1. What will objects look like in different dimensions?

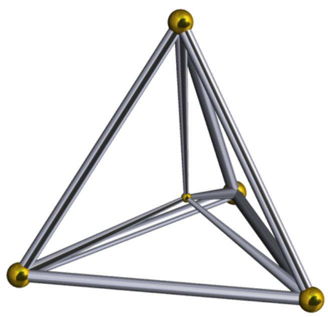
In 3 spatial dimensions, to form a platonic solid, at least three 2D regular polygons need to be around an angle, or else the resulting shape would not need to be in 3D. However, if the shapes equate or exceed 360 degrees, i.e. a flat shape or a shape that does not fit around a point in 2D, it would not form a regular polygon.

Similarly, in 4 dimensions, 3 platonic solids need to be around a shared edge. However, this time we look at their dihedral angle, which is the angle of their edges. However, their angles must be lower than 360 degrees.

There are 6 Convex Regular Polyhedra in 4 spatial dimensions

The 5-cell: 4-Simplex

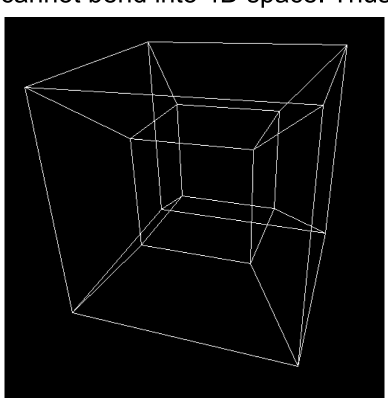
This is created by putting 3 tetrahedrons on a shared edge. As each tetrahedron has a dihedral angle of 70.5 degrees, we can definitely form a valid corner. If we put 3 tetrahedrons on every edge, we form a 4-simplex. The 4 just represents the number of dimensions of the shape. This will only apply to the simplex, the cube and the orthoplex. There are 5 tetrahedrons in total, 1 on each face, and the outer triangle.



The 8-cell: 4-Cube

This is created by putting 3 cubes on a shared edge. As each cube has a dihedral angle of 90 degrees, 3 of them would form 270 degrees, which is a valid corner. If we put 3 cubes on every edge, we form a hypercube, tesseract or a 4-cube. There are 8 cubes in the hypercube. The 6 cubes on each face, the inner cube and the outer cube.

However, we cannot add another cube as that would form 360 degrees, making a flat 3D shape which cannot bend into 4D space. Thus, there is only one polychoron a cube can form.

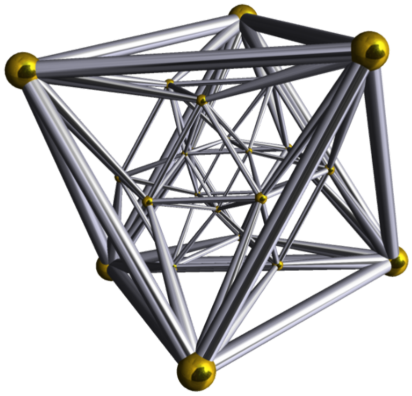


The 16-cell: 4-orthoplex

This is created by putting 4 tetrahedra along a shared edge. As each tetrahedra has a dihedral angle of 70.5 degrees, it is possible to form a valid 4D corner. This will form the 16-cell, or the 4-orthoplex. There are a total of 16 tetrahedra in this shape. The shapes get more and more complex the further we go and later it becomes harder and harder to tell where the 3D shapes are.

The 24-cell: Polyoctahedron

This is formed by putting 3 octahedra along a shared edge. As each octahedra has a dihedral angle of 109.5 degrees, it is possible to form a valid 4D edge. This shape is the polyoctahedron, or the 24-cell.



However, we cannot add another octahedron to a shared edge as that would be 438 degrees, more than 360 degrees, which cannot form a valid 4D corner. Thus, there is only 1 polychoron that the octahedron can form.

The 120-cell: Polydodecahedron

This is formed by putting 3 dodecahedra along a shared edge. As each dodecahedron has a dihedral angle of 116.5 degrees, it is possible to form a valid 4D corner. This results in a shape called the polydodecahedron, or the 120-cell. This shape has 120 dodecahedra in it.

The 600-cell: Polytetrahedron

This is formed by putting 5 tetrahedra along a shared edge. As each tetrahedra has a dihedral angle of 70.5 degrees, the angle sum surmounts to 352.5, which is possible but has very little bending. This results in a shape that has a large amount of tetrahedra, namely 600. It is called the polytetrahedron, or 600-cell. Most of them are crunched up in the middle.

However, we cannot add another tetrahedra to a shared edge as that would be 423 degrees, which is above 360 degrees, which cannot form a valid 4D corner. Thus, there are 3 polychorons that the tetrahedra can form.

There are 3 Convex Regular Polyhedra in 5 spatial dimensions

5-simplex, 5-cube and 5-orthoplex.

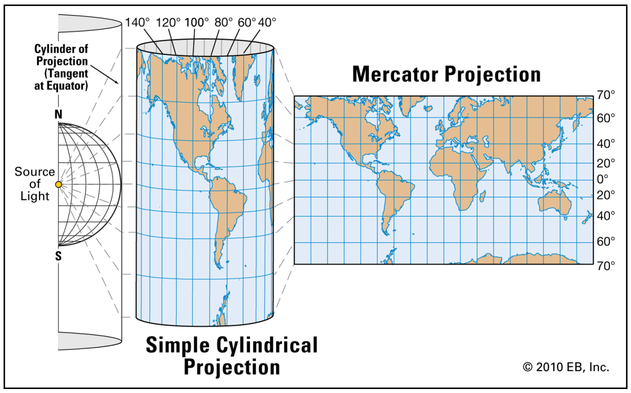
This continues infinitely.

The simplex series was proved to have a small enough angle to go on infinitely, while the cube always has a dihedral angle of 90 degrees, so it always works. There is always the dual of the cube, which is replacing every face with a vertex and every vertex with a face. This results in the orthoplex series.

From 3d to 2d:

A good example of this would be the Mercator projection of the Earth. The Mercator projection involves fitting the globe into a cylinder and projecting the globe onto the cylinder, in the process distorting the areas near the poles. Enlarging countries such as Canada and Russia while diminishing countries such as Brazil and Australia.

The Earth cannot be projected onto a cylinder properly without distortion of certain parts because there is a change in dimensions.



From the Mercator Projection example, we can see that higher dimensional objects will be distorted when presented in a lower dimension.

2. How will varying the number of dimensions affect the object?

We cannot see objects in higher dimensions.
However, we can imagine them.
We can't see the 4th dimension. Imagine this: if we are 2-dimensional instead of 3-dimensional, we would not be able to see most of the objects in the third dimension as we are stuck in a world which has no height. Thus, similarly, physicists concluded that we cannot see the 4th dimension.

In addition, a creature in the 1st dimension can only move forwards and backwards, and it is also unable to move sideways. A creature in the 2nd dimension has a new parameter, the breadth. With this new parameter, it can move sideways, forwards and backwards. However, it cannot move upwards or downwards. Creatures which belong to the third dimension also have a new parameter, height. Hence, creatures in the third dimension can not only move forwards backwards and sideways, they can also move upwards and downwards. Hence, using this, scientists realised that a 2-dimensional space is composed of infinitely many 1-dimensional spaces stacking in every direction(breadth and length). Similarly, a 3-dimensional space is also composed of infinitely many 2-dimensional spaces stacked together in three directions(the width, length and height). Hence, scientists concluded that a 4-dimensional space should be composed of infinitely many 3-dimensional spaces stacking in all 4 parameters of the 4th dimension.

Hyperspheres

For 2-dimensional Euclidean space the hypersphere is circle (1-sphere),
For 3-dimensional Euclidean space the hypersphere is sphere (2-sphere).
For 4-dimensional Euclidean space the hypersphere is glome (3-sphere).

$$\mathbf{V} = \frac{1}{2}\pi^2 R^4$$

A hypersphere in 5-space (also called a 4-sphere due to its surface being 4-dimensional) consists of the set of all points in 5-space at a fixed distance r from a central point P. The hypervolume enclosed by this hypersurface is

$$V = \frac{8\pi^2 r^5}{15}$$

The volume of six-dimensional space bounded by this 5-sphere is

$$V_6 = \frac{\pi^3 r^6}{6}$$

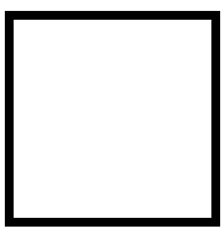
The volume of the space bounded by this 6-sphere is

$$V_7 = \frac{16\pi^3 r^7}{105}$$

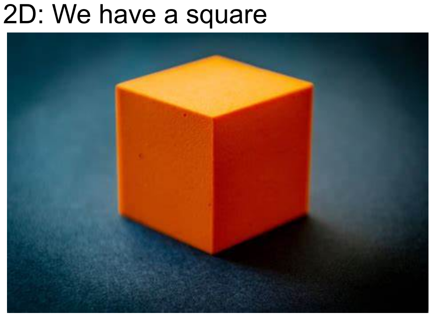
The volume of the space bounded by this 7-sphere is

$$V_8 = \frac{\pi^4}{24} R^8$$

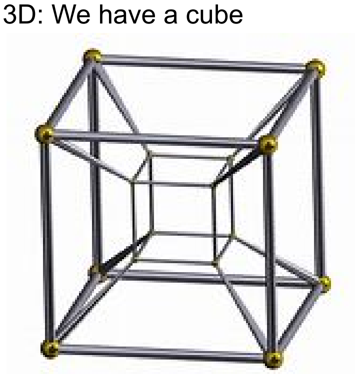
Hypercubes



2D: We have a square



3D: We have a cube



4D: We have a tesseract

And so on…

			m	0	1	2	3	4	5	6	7	8	9	10
n	n-cube	Names	Schläfli Coxeter	Vertex 0-face	Edge 1-face	Face 2-face	Cell 3-face	4-face	5-face	6-face	7-face	8-face	9-face	10-face
0	0-cube	Point Monon	( ) ●	1										
1	1-cube	Line segment Dion[4]	{} ◉	2	1									
2	2-cube	Square Tetragon	{4} ◉ 4 ●	4	4	1								
3	3-cube	Cube Hexahedron	{4,3} ◉ 4 ●–●	8	12	6	1							
4	4-cube	Tesseract Octachoron	{4,3,3} ◉ 4 ●–●–●	16	32	24	8	1						
5	5-cube	Penteract Deca-5-tope	{4,3,3,3} ◉ 4 ●–●–●–●	32	80	80	40	10	1					
6	6-cube	Hexeract Dodeca-6-tope	{4,3,3,3,3} ◉ 4 ●–●–●–●–●	64	192	240	160	60	12	1				
7	7-cube	Hepteract	{4,3,3,3,3,3}	128	448	672	560	280	84	14	1			

		Tetradeca-7-tope	◉–4–●–●–●–●–●–●											
8	8-cube	Octeract Hexadeca-8-tope	{4,3,3,3,3,3,3} ◉–4–●–●–●–●–●–●–●	256	1024	1792	1792	1120	448	112	16	1		
9	9-cube	Enneract Octadeca-9-tope	{4,3,3,3,3,3,3,3} ◉–4–●–●–●–●–●–●–●–●	512	2304	4608	5376	4032	2016	672	144	18	1	
10	10-cube	Dekeract Icosa-10-tope	{4,3,3,3,3,3,3,3,3} ◉–4–●–●–●–●–●–●–●–●–●	1024	5120	11520	15360	13440	8064	3360	960	180	20	1

3. What is the difference between 3D+2D and 5D?/ 3D+1D and 1D, etc? Are there any formulas to calculate objects of 3D+2D or 3D+1D?

4D contains 3 temporal dimensions and 1 spatial dimensions.This is why 4D is also 3D+1D

5D contains 3Temporal dimensions and 2 spatial dimensions.This is why 5D is also 3D+2D

Note that the 3temporal dimensions are length × width × height

While the spatial dimensions are related to time,1D of spatial dimensions is time while 2D is time × timeline

The set of points in Euclidean 4-dimension having the same distance R from a fixed point P. The hyper-volume of the enclosed space is:

$$\mathbf{V} = \frac{1}{2}\pi^2 R^4$$

A hypersphere in 5d consists of the set of all points in 5-dimension at a fixed distance r from a central point P. The hypervolume enclosed by this hypersurface is:

$$V = \frac{8\pi^2 r^5}{15}$$

How can we represent 4D and 5D geometrically?

In 0D there is only a point. To get to the next dimension, we need to double the point.
For example, in 1D, there are 2 points that form a line.
In 2D, there are 4 points that form an image.
So, there will be 16 points that form an object in 4D and 32 points that form an object in 5D.

8. References where did u guys find ur links

11 Dimensions Explained (Eleven Dimensions) - What are Dimensions & How Many Dimensions are There - YouTube
Q: What would life be like in higher dimensions? | Ask a Mathematician / Ask a Physicist
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