Charging Ahead!

Singapore's 2030 Electric Vehicle Charging Stations Numbers and Distribution in 2030 based on Bass Diffusion Model and Graph Theory

Final Written Report

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1) Introduction

In February 2021, Singapore Government announced its plans to increase the number of Electric Vehicle (EV) charging stations (CS) to 60,000 by 2030. While the plan to accelerate the adoption of EVs is to be lauded, the Government did not provide the basis of its 60,000 CS projection and the distribution of these CS across Singapore. As a result, concerns were raised in the parliament that the Government may be "over-building" CS by 2030 (Tan, 2014).

Currently, there are no published studies of EV adoption in Singapore based on innovation diffusion models. Hence, this study aims to utilize the Bass Diffusion Model to project the number of CS required in Singapore in 2030 and Graph Theory's Dijkstra's Algorithm to distribute the CS optimally.

1.1 Research Questions

Question 1: Which diffusion model can be applied to project Singapore's EV population and CS requirements?

Question 2: What is Singapore's EV and CS population in Year 2030 based on the selected diffusion model?

Question 3: What is the optimal distribution of CS in Singapore based on charging demand using Poisson distribution?

Question 4: How can residents in Singapore find the nearest charging station around them?

2) Literature Review

Gnann et al. (2018) compared 40 EV market diffusion models from 16 countries which were published after 2010. Studies done for the Singapore EV market are glaringly missing.

The Bass diffusion model remains the predominant model for adoption study due to the simplicity of the model and the ability to generalise the model for various products (NZ Transport Agency (2018)). A number of transport studies have also deployed the Bass model successfully (Massiani and Gohs, 2015). This study will utilise the Bass model for projecting Singapore's 2030 EV and CS population.

According to Zhang (2019), the number of EVs that come to recharge each day satisfies the Poisson distribution while the charge of a single electric car meets the normal distribution. Using these distributions, CS numbers and installation can be planned for specific subzone.

Dijkstra's algorithm can be used to determine the shortest distance between nodes in a graph and therefore is useful for the study to find the shortest path between CS. Studies have also provided methods to find the shortest path to complicated networks (Akram, 2021).

3) Methodology

3.1 Bass Model Parameter/Coefficients Estimation

The Bass model is a Riccati differential equation with constant coefficients that describes the process of how new products get adopted in a population (equation 1). The model presents a rationale of how current adopters and potential adopters of a new product interact.

$$n_t = \frac{dN_t}{dt} = p(M - M_t) + q\frac{1}{M}N_t(M - N_t)$$

With:

n_t: product purchases in period t

N_t: cumulative product purchases until beginning of period t

M: cumulative market potential on the whole product's life cycle

p: coefficient of innovation

q: coefficient of imitation

Rearranging the equation above:

$$S_t = a + bY_{t-1} + cY_{t-1}^2$$

With:

$S_t = Y_t - Y_{t-1}$ is the new adoption in time t

$a = pm$

$b = q - p$

$c = -\frac{q}{m}$

Using the set of three regression coefficients (a, b, c), it is possible to identify the set of three Bass Model parameters (p, q, m).

To estimate p and q, the regression equation above can be estimated using analogous products or other markets that have similar features to the new product. The M (market potential) for Singapore is projected to be 650,000 EV in 2050 based on the current car growth rate and the Government's intention to have only clean-energy cars for sale from 2030.

3.2 Adoption Curve based on Bass Model

Using the estimated p and q, the EV Bass adoption curve similar to the below can be produced:

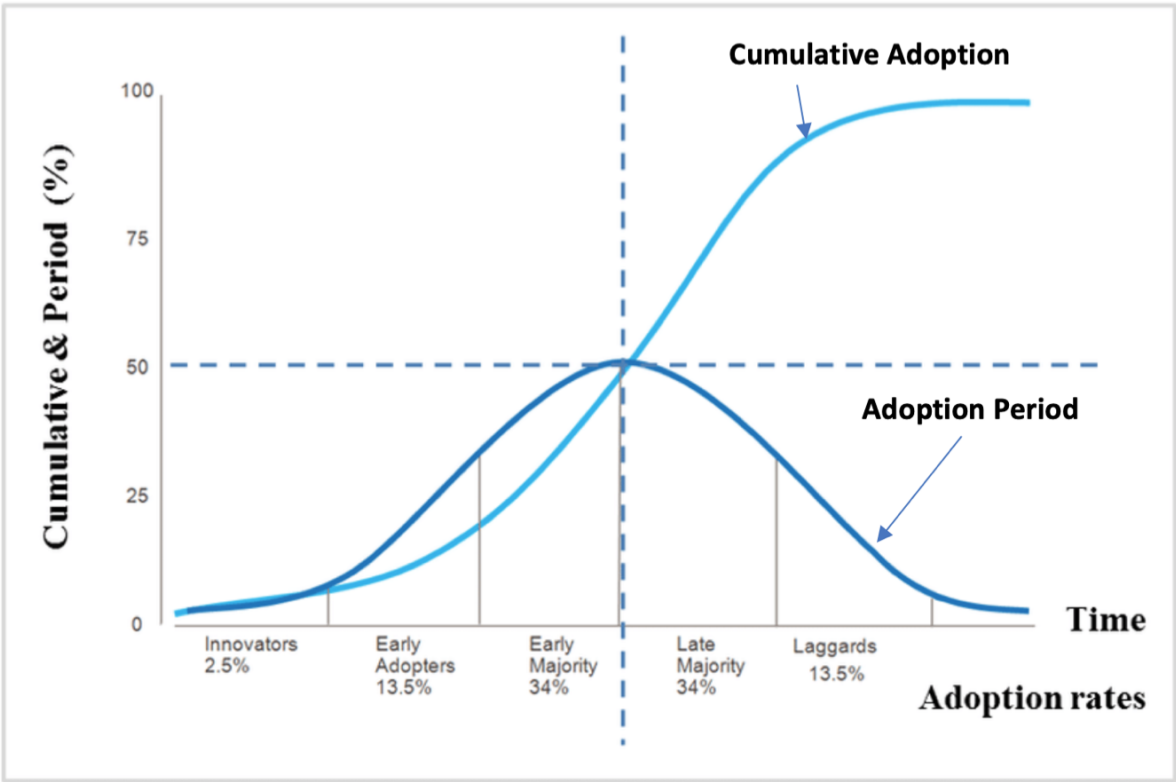

Figure 1: Bass Adoption Curve

3.3 CS Requirements and Assumptions

With the projected 2030 EV population, the following assumptions are applied to calculate the number of CS required in Singapore:

Table 1: Assumptions for the Calculations of Number of Charging Stations

Key Assumptions		Source
Annual Mileage Per Car (KM)	17,500	Source: https://data.gov.sg/dataset/annual-mileage-for-private-motor-vehicles
Average Consumption per EV (KWh/km)	0.2	Wolbertus and Hoed (2020)
% of Total Power for Slow Charging	90%	Wolbertus and Hoed (2020); Funke et al. (2019)
% of Total Power for Fast Charging	10%	Wolbertus and Hoed (2020); Funke et al. (2019)

3.4 Distribution of CS

The estimated CS number from the Bass Model consists of two types: residential and non-residential CS.

Using Shanghai's CS percentage in public parking lots as a proxy for Singapore, this study assumes 31% of the charging spaces are equipped with CS and the remaining CS are residential stations distributed in HDB.

Regression analysis has also shown that the number of CS is only significantly related to population.

```
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        1248.9888   558.9908   2.234   0.1550
Population            0.8411     0.1019   8.258   0.0143 *
Income               21.4006     6.2680   3.414   0.0761 .
Traffic.congestion -103.1957    35.5368  -2.904   0.1009
```

Therefore, the CS distribution for Singapore can be based on the below assumptions:

$$\textit{Number of non-residential charging stations} \geq 31\% \times \textit{Parking lots,}$$

And

$$\textit{Number of residential charging stations} \propto \textit{Population density}$$

Accordingly, the number of CS allocated to each subzone is given by:

$$\text{CS} = (k_1 \times P) + (k_2 \times pop \times C)$$

with:

k_1: Percentage of public parking space equipped with CS;
k_2: Average number of EV per capita;
P: Number of parking lots;
pop: Population in subzone;
C: Number of CS needed for each EV

For the above, the study will utilise Urban Redevelopment Authority (URA) carpark distribution data and population density data in Singapore. 271 out of 300 subzones in Singapore with adequate data are available for this research.

Shi et al (2020) discussed the number of cars arriving at a public car park (non-residential) can be considered as a random variable subject to both Binomial Expansion and Poisson distribution. Poisson distribution equation will be used to assess the availability of CS for each Singapore subzone (see Appendix D).

3.5 Shortest Distance to CS

Graph Theory is applied to study the path to the nearest CS given a point in Singapore, and Dijkstra's Algorithm is used to find the shortest path.

4) Results

4.1 Research Question 1

Given the advantages of using the Bass model for projecting EV adoption (Zhu et. al 2018) and its wide use in the transport sector, this study will apply the Bass model to forecast EV population in Singapore for 2030.

4.2 Research Question 2

The study examined analogous products such as diesel car and mobile phone adoption in Singapore, and also overseas markets to estimate the Bass coefficients for Singapore. Regression results are shown in Table 2 and Appendix B. Given the statistical significance of the Singapore Diesel Car and Mobile Phone regressions, the study decided to use these two results to estimate its average Bass coefficients.

Table 2 Bass Model Coefficients Estimation and Comparison

	Bass Model Coefficients		
	p	*q*	Sources
Singapore Diesel Car Adoption	0.000856	1.111246	Based on LTA data
Singapore Mobile Phone Adoption	0.00567	0.42	Based on World Bank data sets
Singapore Average	**0.00326**	**0.76576**	
US EV Adoption	0.0148	0.5676	Kong and Bi (2014)
Japan EV Adoption	0.0032	0.4189	Kong and Bi (2014)
China EV Adoption	0.00007366	0.5902	Xing (2019)
Beijing EV Adoption	0.001	0.304	Zhu and Du (2018)
Malaysia New Car Adoption	0.0503	0.484	Ismail and Abu (2013)
Moroco EV Adoption	0.01852	0.1151	Ayyadi and Mohamed (2019)
Toyota Prius (US) Adoption	0.001645368	1.455182	Massiani and Gohs (2015)
Civic Hybrid (US) Adoption	0.003437559	0.631287	Massiani and Gohs (2015)
Ford Escape (US) Adoption	0.03673027	0.432231	Massiani and Gohs (2015)
HEV Japan/Korea	0.000618	0.8736	Massiani and Gohs (2015)
Electric (Germany) Adoption	0.0019	1.2513	Massiani and Gohs (2015)
LPG (Germany) Adoption	0.0779	0.3718	Massiani and Gohs (2015)
CNG (Germany) Adoption	0.1187	0.0349	Massiani and Gohs (2015)
New Zealand EV Adoption	0.0169	0.765	NZ Transport Agency (2018)
Norway EV Adoption	0.0023	0.58	Regression based on 2010 to 2020 data
World Average	**0.02320**	**0.59151**	

The projections of EV in Singapore from 2020 to 2050 based on the Singapore Average Bass coefficients are in Table 2 and Table 3:

Table 3: Singapore EV Projections from 2015 to 2050

M	649,720
p	0.003263095
q	0.77

S(t) = p*M + (q-p)*Y(t-1) - (q/M)*Y(t-1)^2

	Sales Per Year	Singapore Average P Q	Singapore Mobile P Q	Singapore Diesel Car P Q
2015	1	1	1	1
2016	11	12	12	12
2017	302	314	314	314
2018	246	560	560	560
2019	560	1,120	1,120	1,120
2020	97	1,217	1,217	1,217
2021	3,046	4,263	5,405	3,122
2022	5,349	9,613	11,311	7,127
2023	9,341	18,954	19,602	15,510
2024	16,149	35,102	31,165	32,877
2025	27,433	62,536	47,142	68,091
2026	45,194	107,730	68,934	136,325
2027	70,585	178,315	98,125	256,469
2028	100,609	278,924	136,264	429,305
2029	123,105	402,029	184,432	591,335
2030	**118,172**	**520,201**	**242,580**	**650,435**
2031	79,832	600,033	308,774	649,639
2032	35,301	635,333	378,805	649,729
2033	10,820	646,153	446,723	649,719
2034	2,728	648,881	506,533	649,720
2035	644	649,525	554,260	649,720
2036	150	649,675	589,026	649,720
2037	35	649,710	612,495	649,720
2038	8	649,718	627,454	649,720
2039	2	649,719	636,618	649,720
2040	**0**	**649,720**	**642,087**	**649,720**
2041	0	649,720	645,301	649,720
2042	0	649,720	647,170	649,720
2043	0	649,720	648,252	649,720
2044	0	649,720	648,876	649,720
2045	0	649,720	649,235	649,720
2046	0	649,720	649,441	649,720
2047	0	649,720	649,560	649,720
2048	0	649,720	649,628	649,720
2049	0	649,720	649,667	649,720
2050	**0**	**649,720**	**649,690**	**649,720**

The EV population is projected to reach 520,201 in 2030 and hit the market potential of 649,720 in 2035. Given the push to sell only green energy cars from 2030 and the push to adopt EV in 2021, it is entirely possible to only have EV in Singapore from 2035.

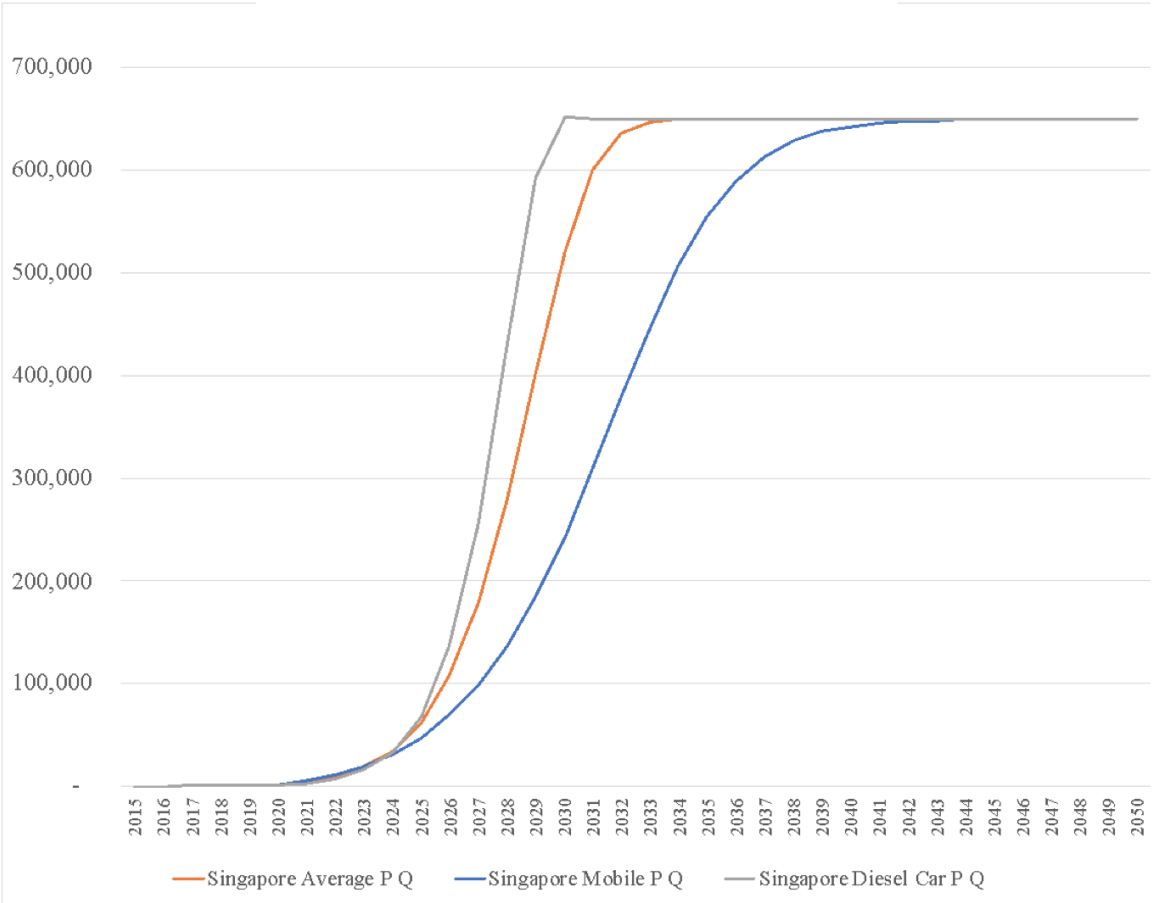

Figure 2: EV Population in Singapore from 2015 to 2050

Based on the assumptions in Table 1, Table 4 shows the number of CS required for Singapore in 2030 is 94,828.

Table 4: Estimation of Total Charging Stations in Singapore for Year 2030

Electric Vehicle Population 2030	520,201
Key Assumptions	
Annual Mileage Per Car (KM)	17,500
Average Consumption per EV (KWh/km)	0.20
% of Total Power for Slow Charging	90%
% of Total Power for Fast Charging	10%
Total Mileage for Singapore EV 2030 (km)	9,103,511,470
Total Power Required for EV in 2030 (KWh)	1,820,702,294
Annual % of Total Power for Slow Charging	90%
Annual Total Power for Slow Charging (KWh)	**1,638,632,065**
Annual % of Total Power for Fast Charging	10%
Annual Total Power for Fast Charging (KWh)	**182,070,229**

	Power Level (KW)	Efficiency (%)	Power Capacity per day	Time for one charge (hours)	Number of Charges per Day	Total Charge Hour per day (Hr)	Total Power Capacity Produced Per Day Per Station (KWh)	Total Power Capacity Required Per Yeasr Per Station (KWH)
AC Slow, Type 1, Single Phase	3.3	0.95	75.24	7.6	2	15.2	47.652	17,393
AC Medium, Type 2, Single Phase	6.6	0.9	142.56	3.8	4	15.2	90.288	32,955
DC Fast Charging	50	0.9	1080	0.6	30	18	810	295,650

Source: LTA, E-Mobility Technology Roadmap

Slow Charging Station (KWh) in 2030	94,212
Fast Charging Station (KWh) in 2030	616
Total Charging Stations in Singapore Year 2030	**94,828**

4.3 Research Question 3

4.3.1. Distribution of CS

Based on Section 3.4, Python is used to fetch and calculate the number of CS in each subzone. OpenStreetMap API is used to find the coordinates of each parking and subzone, and Tableau to plot the graph of the non-residential and residential CS distributions (Figure 3 to 5). The result data can be found in Appendix C.

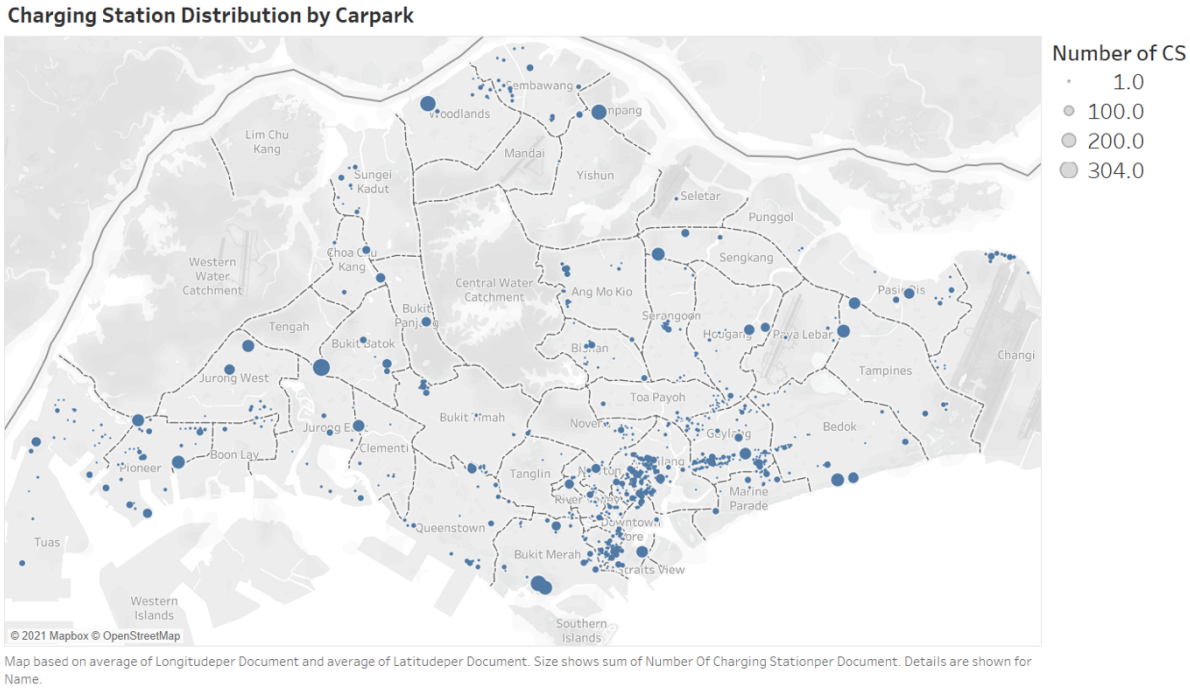

Figure 3: Non-residential CS Distribution

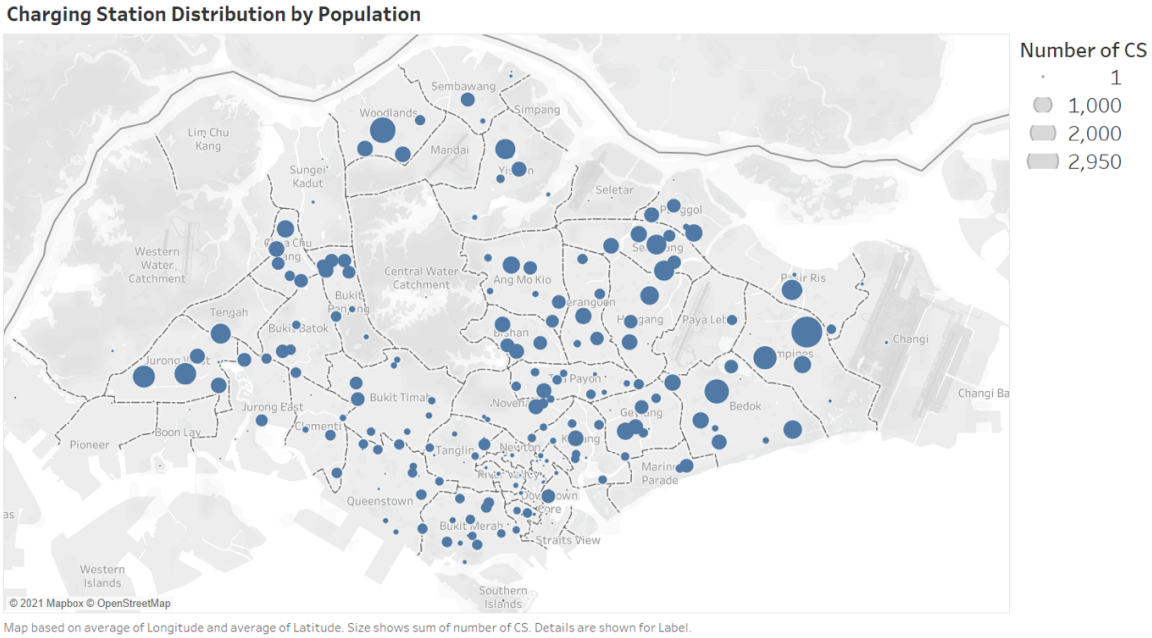

Figure 4: Residential CS Distribution

By merging Figure 3 & 4, the combined distribution of CS is shown in Figure 5.

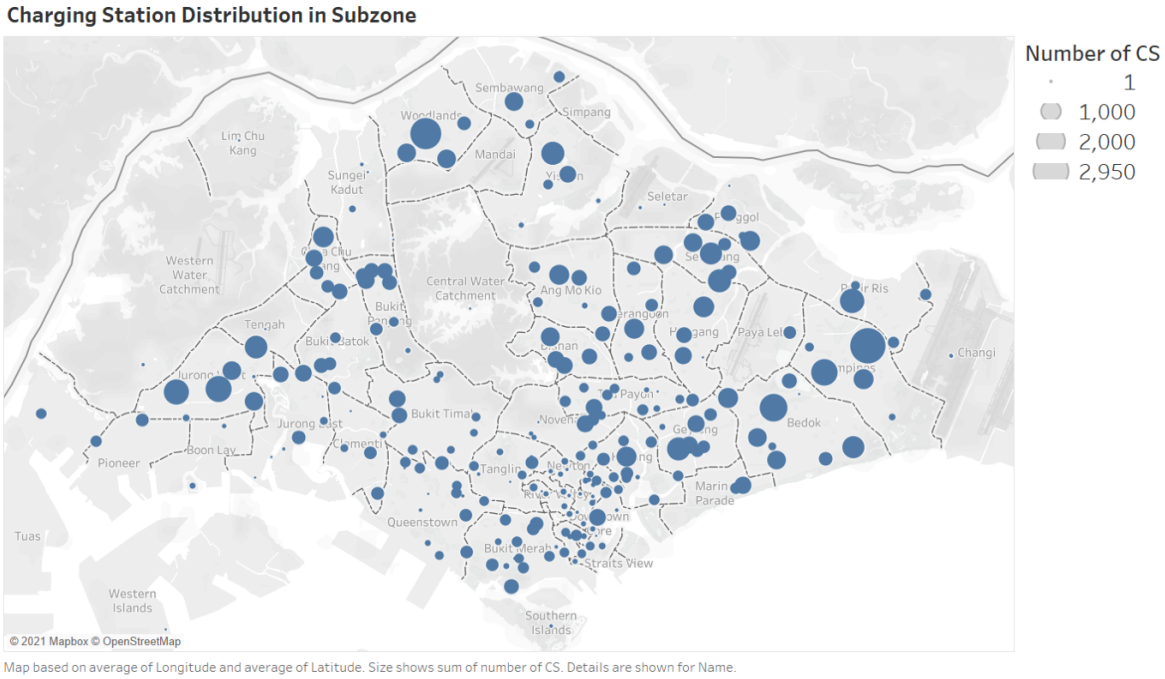

Figure 5: Combined CS Distribution

4.3.2. Availability of CS in subzones

Based on derivation in Appendix D, Cumulative Poisson Probability is used in the study to get the probability $P(x \leq k)$ that x number of CS will be larger than the demand λ, meaning that no cars need to wait for the CS if they need in each subzone:.

$$P(x \leq k) = \sum_{k=0}^{x} \frac{\lambda^k}{k!} \times e^{-\lambda}$$

Accordingly, the probability that the CS in a subzone for charging can exactly satisfy the EV that comes to charge is calculated using Python and plotted by Tableau:

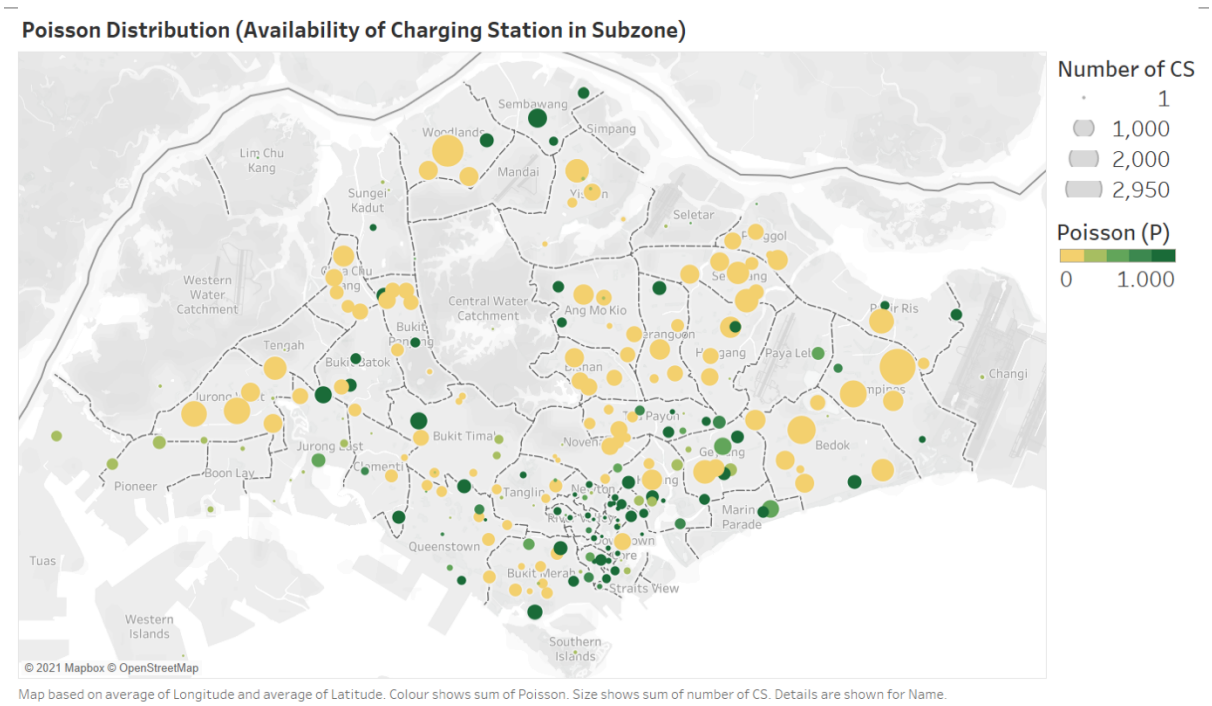

Figure 6: Availability of CS in Subzones

4.4 Research Question 4

Based on the CS distributions, the study then employed Graph Theory and Dijkstra algorithm to determine the shortest path for an EV. The detailed steps taken are listed in Appendix E.

As an example, at starting point location of (1.2849587, 103.813504,17) and the estimated CS location (1.2910574,103.823168), applying the Dijkstra's Algorithm provides the shortest distance as 2.047km (Figure 7) , while Google Maps gives the results as 2.1km (Figure 8). As the two give different paths, Dijkstra's algorithm provides the shortest path.

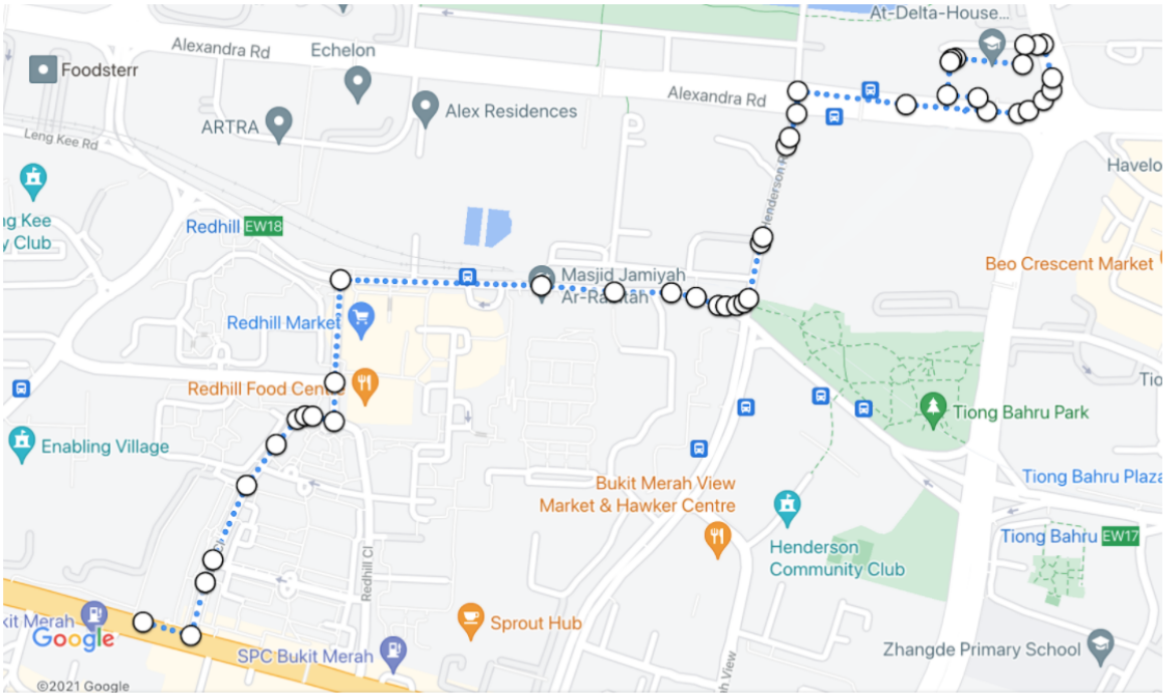

Figure 7: Results using Dijkstra's Algorithm (2.047km)

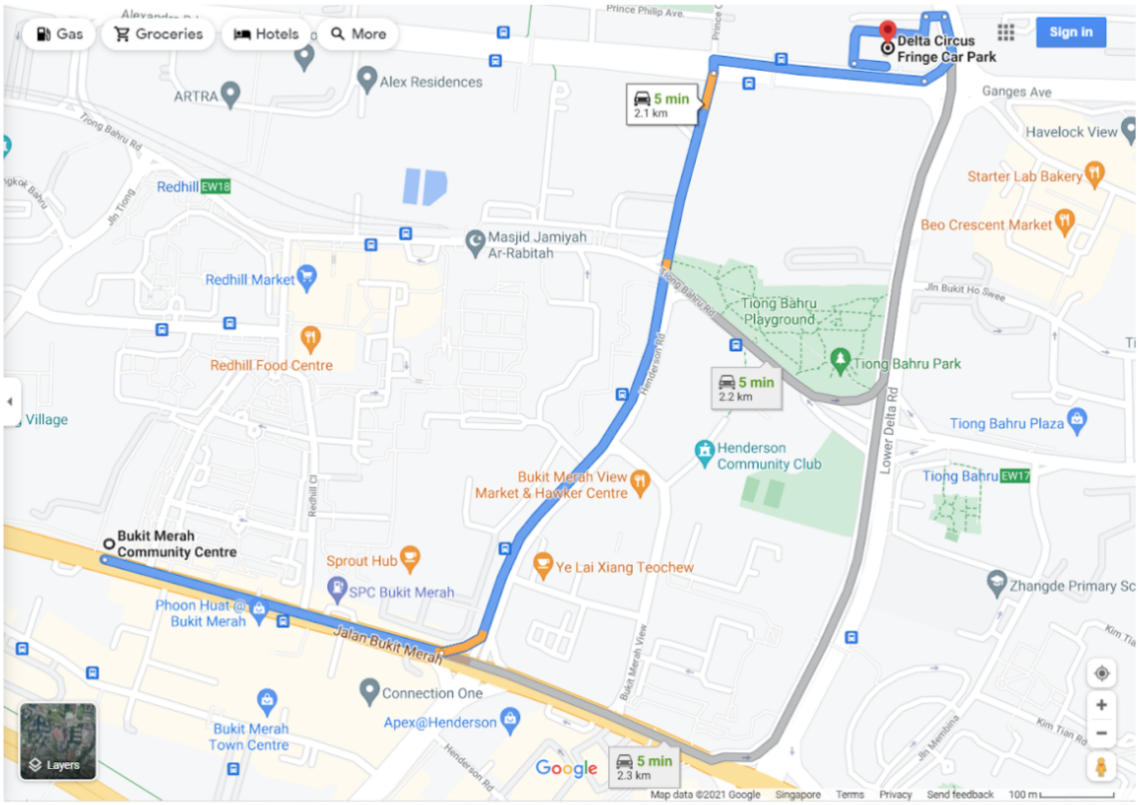

Figure 8: Results using Google Map (2.1km)

The algorithm in Appendix E can be replicated using any starting and CS distribution locations to find the shortest path.

5) Conclusion and Limitations

The number of charging stations required in 2030 is projected to be higher than Singapore's plan of 60,000. The government may consider accelerating the CS deployment to ensure the shortfall will not impact EV demand in 2030.

The 94,828 CS projected can be distributed using URA subzone data by dividing them into residential and non-residential uses. A combination of Binomial Distribution and Poisson distributions is used to evaluate the availability of CS for EV in this subzone. With the use of distribution maps, the path to the nearest CS can be determined using Graph theory and Dijkstra's algorithm.

The limitations and extension of the study are:

1) The Bass coefficient estimations can be more accurate with better/wider data collection and/or conducting a survey to determine buyers' adoption of future EV;
2) Efficiency of CS is assumed to be constant through the years with no improvement or degradation. Any change to the efficiency will impact the number of CS required.
3) Application of other algorithms such as novel search algorithms or multi-objective optimization models can be applied for better understanding of CS distribution in Singapore.

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Appendix A: Data Sources

The data for this research were collected from the following sources:

1) Land Transport Authority:
    a. Motor Vehicle Population by Type of Fuel Used from 2001 to 2020
    b. https://www.greenplan.gov.sg
    c. https://www.lta.gov.sg/content/ltagov/en/industry_innovations/technologies/electric_vehicles.html

2) Data.gov.sg:
    a. Household projections
    b. White goods penetration in Singapore
    c. Annual Mileage Per Car

3) World Bank Data
    a. Mobile phone historical penetration numbers from 1998

4) Urban Redevelopment Authority
    a. Availability of car parks: https://www.ura.gov.sg/maps/api/#car-park-available-lots
    b. Zones:
        i. https://data.gov.sg/dataset/master-plan-2019-subzone-boundary-no-sea
        ii. https://www.ura.gov.sg/maps/?service=demographics

Appendix B: Regression Results for Calculation of *p* and *q*

Singapore Diesel Car Bass Model Coefficients Estimation

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.993234989
R Square	0.986515744
Adjusted R Square	0.984929361
Standard Error	188.2750945
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	44087117.51	22043558.75	621.8647796	1.2692E-16
Residual	17	602607.6908	35447.51123		
Total	19	44689725.2			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	16.25645877	51.9173505	0.313121887	0.75799697	-93.27957606	125.7924936	-93.27957606	125.7924936
X Variable 1	1.110390042	0.031885856	34.82390531	3.00985E-17	1.043116765	1.177663318	1.043116765	1.177663318
X Variable 2	-6.21309E-05	1.89366E-06	-32.80996115	8.17057E-17	-6.61261E-05	-5.81356E-05	-6.61261E-05	-5.81356E-05

p	0.000856
q	1.11

Singapore Mobile Phone Bass Model Coefficients Estimation

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.842714717
R Square	0.710168094
Adjusted R Square	0.673939106
Standard Error	123887.6273
Observations	19

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	6.01715E+11	3.00857E+11	19.60220607	4.97931E-05
Residual	16	2.4557E+11	15348144194		
Total	18	8.47285E+11			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	25238.40062	44012.82139	0.573432919	0.574323868	-68064.61267	118541.4139	-68064.61267	118541.4139
Y (t-1)	0.41460043	0.086604371	4.787292216	0.000201481	0.231007366	0.598193495	0.231007366	0.598193495
Y (t-1) Squared	-8.69795E-08	2.37326E-08	-3.664985204	0.002091456	-1.3729E-07	-3.66687E-08	-1.3729E-07	-3.66687E-08

p	0.00567
q	0.42

Norway Bass Model Coefficients Estimation

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.984518117
R Square	0.969275922
Adjusted R Square	0.961594903
Standard Error	7585.384154
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	14521574808	7260787404	126.1910519	8.91077E-07
Residual	8	460304422.1	57538052.77		
Total	10	14981879231			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	6465.121829	3480.180175	1.857697448	0.100284596	-1560.188046	14490.4317	-1560.188046	14490.4317
X Variable 1	0.575286506	0.064920369	8.861417703	2.07676E-05	0.425579867	0.724993146	0.425579867	0.72499315
X Variable 2	-8.73118E-07	1.79699E-07	-4.858794268	0.001257727	-1.2875E-06	-4.58733E-07	-1.2875E-06	-4.5873E-07

p	0.0023
q	0.58

Appendix C: Data for CS Distribution

As the data for the detailed charging station distribution in each subzone is very large, links will be provided for further reference.

1. Residential CS distribution by population: https://github.com/sileneer/pw-2021/blob/master/distribution/population_distribution.json

   The data will be in the format of:

Label of subzone	Population	Latitude	Longitude	Number of CS

2. Non-residential CS distribution by carpark: https://github.com/sileneer/pw-2021/blob/master/distribution/carpark_distribution.json

   The data will be in the format of:

Name	Coordinates	Count (of parking lots)	Latitude	Longitude	Number of CS	Subzone

3. Overall CS distribution in subzones and the corresponding Poisson Probability: https://github.com/sileneer/pw-2021/blob/master/distribution/cs_in_subzone.json

   The data will be in the format of:

Name (of subzone)	Population	Number of CS	Latitude	Longitude	Poisson

The codes to fetch and process data from online sources are also provided in the following repository: https://github.com/sileneer/pw-2021

Appendix D: Poisson Distribution for CS in Each Subzone

Shi et al (2020) discussed the number of cars arriving at a parking lot in a public area (non-residential) can be considered as a random variable subject to Poisson distribution with probability distribution defined by:

$$P(x = k) = \frac{\lambda^k \times e^{-\lambda}}{k!} (k = 0, 1, 2, \dots, x)$$

where x is the actual number of CS available in each subzone and λ is the theoretical CS needed for a given population in this subzone.

By Binomial Distribution, in time T, given p is the probability that each car is able to charge as soon as come, and k is the number of CS needed when all cars in this subzone come, dividing the T into infinite n parts, the probability P that all cars are able to charge is given by:

$$P = \lim_{n\to\infty} \binom{n}{k} p^k (1-p)^{n-k}$$

Given λ as the expected CS needed,

$$P = \lim_{n\to\infty} \binom{n}{k} p^k (1-p)^{n-k} = \lim_{n\to\infty} \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k}$$

Upon expanding,

$$\begin{aligned} P &= \lim_{x\to\infty} \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k} \\ &= \lim_{n\to\infty} \frac{n(n-1)(n-2)\dots(n-k+1)}{k!} \times \frac{\lambda^k}{n^k} \left(1 - \frac{\lambda}{n}\right)^{n-k} \\ &= \lim_{n\to\infty} \frac{\lambda^k}{k!} \times \frac{n}{n} \times \frac{n-1}{n} \times \dots \times \frac{n-k+1}{n} \times \left(1 - \frac{\lambda}{n}\right)^{-k} \times \left(1 - \frac{\lambda}{n}\right)^{n} \end{aligned}$$

As

$$\lim_{n\to\infty} \frac{n}{n} \times \frac{n-1}{n} \times \dots \times \frac{n-k+1}{n} \times \left(1 - \frac{\lambda}{n}\right)^{-k} = 1$$

and

$$\lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^{n} = e^{-\lambda}$$

Hence,

$$P(x = k) = \frac{\lambda^k}{k!} \times e^{-\lambda}$$

Appendix E: Steps for Application of Dijkstra's Algorithm

The following steps were applied to determine the nearest CS for Singapore EVs.

1. Model the map:
    a. Given a point on the map, follow the road until finding the crossing and record the distance between the point and the crossing.
    b. From each crossing, follow the road until finding the crossing, and record the distance between the previous crossing and the next crossing.
    c. Repeat this process until we have searched all the crossings in a certain range

2. Apply Dijkstra's Algorithm:
    a. Mark every node as unvisited.
    b. Find the unvisited node connected with the starting point. Calculate the distance to each node passing through the current node, and record the smallest distance and node.
    c. Update the smallest distance to its neighbouring nodes from the starting point, and mark the current node as visited
    d. Repeat this process until all the nodes are visited. The shortest path is the recorded nodes.