

Group 8-26

Poker Probability

Group leader

Jayden Teo (1O125)

Group members

Tan Yi Han (1O124)

Bryan Wong (1O129)

Matthew Yeo (1O131)

Table of Contents

1 Project description	3
1.1 Objectives	3
1.2 Research Questions	3
2 Literature Review	4
3 Results and findings	5
3.1 Research Question 1	5
3.2 Research Question 2.....	7
3.3 Research Question 3	9
4 Conclusion	11
5 References	12

1 Project description

The goal of our project is to find out the sum of cards when drawn at random, and to find the probability of the sum being a multiple of 2, 3, 5 or a prime number. We use Excel to help with our calculations. Essentially, we will pick out n number of poker cards, add the values of the poker cards up. We will then check if the total value is a multiple of a chosen number.

1.1 Objectives

The objective of our project is to find out the sum of n random numbers and find the probability of the sum being a multiple of 2, 3, 5 and a prime number.

1.2 Research questions

- 1 One will draw n cards from a deck of cards with values of 1 to 9. This question considers all four suits (i.e., each digit can be repeated up to 4 times)
 - (i) Suppose $n = 4$, we will draw 4 cards and find the probability of the sum of 4 numbers being a prime number or multiple of 2, 3 or 5.
 - (ii) Suppose $n = 5$, we will draw 5 cards instead and find the sum of the 5 cards, then find the probability of the sum being a prime number or multiple of 2, 3 or 5.
- 2 One will draw n cards from a single suit of a deck of cards with values of 1 to 9. This also means that digits cannot be repeated.
 - (i) Suppose $n = 4$, one would draw 4 cards at random and find the sum. We would then find the probability of the sum being a prime number or multiple of 2, 3 or 5.
 - (ii) Suppose $n = 5$, one would 5 cards at random and find the sum. We would then find the probability of the sum being a prime number or multiple of 2, 3 or 5.
- 3 We would consider drawing four cards which include 10, Jack, Queen and King.
 - (i) We would consider all cards in a poker deck - including 10, Jack, Queen and King.
 - (ii) We would consider a single suit and all cards including 10, Jack, Queen and King.

2 Literature review

A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal, and Pierre de Fermat. (Apostol, 1969) This creation led to the solving of many problems, including our project, which we used Google Sheets predominantly. In the document by Google, it shows the exhaustive list of functions in Google Sheets. From there, we found out which functions are appropriate to use for each situation. With knowing each function, we went to learn how to use them, where research by Grubbs (2016) further teaches us how to use different functions in Google Sheets. For our project, we learnt how to use Google Sheets “sum” function to calculate the sum of the different cases. We also had to use Google Sheets “true false” function, where research by Prashanth (2019) teaches us how to, and we went on to use this function to test if the sum of the results is a multiple of 2, 3 or 5. However, this article does not teach us how to test for prime numbers. This led on to us making use of research by Shubham (2019), explaining how to make a custom code in Google Sheets. From there, we managed to create our own code to test for prime numbers. Finally, to find the probability and total number of cases for each situation, we made use of research from Kemp (2015) with his article teaching us how to use Google Sheets “COUNTIF” code which we learnt how to use it to count the number of “trues” in each column in the Google Sheet.

3 Results and findings

3.1 Research Question 1

- (i) We found that the probability of any 4 cards being a multiple of 2 is $\frac{3289}{6577}$, the probability of any 4 cards being a multiple of 3 is $\frac{2201}{6577}$, the probability of any 4 cards being a multiple of 5 is $\frac{1317}{6577}$, and the probability of any 4 cards being a prime number is $\frac{1860}{6577}$.

Number 1	Number 2	Number 3	Number 4	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime Number?
1	1	1	1	4	TRUE	TRUE	FALSE	FALSE
1	1	1	1	2	5	FALSE	FALSE	TRUE
1	1	1	1	3	6	TRUE	TRUE	FALSE
1	1	1	1	4	7	FALSE	FALSE	FALSE
1	1	1	1	5	8	TRUE	FALSE	FALSE
1	1	1	1	6	9	FALSE	TRUE	FALSE
1	1	1	1	7	10	TRUE	FALSE	TRUE
1	1	1	1	8	11	FALSE	FALSE	FALSE
1	1	1	1	9	12	TRUE	TRUE	FALSE
1	1	2	1	5	FALSE	FALSE	TRUE	TRUE
1	1	2	2	6	TRUE	TRUE	FALSE	FALSE
1	1	2	3	7	FALSE	FALSE	FALSE	TRUE
1	1	2	4	8	TRUE	FALSE	FALSE	FALSE
1	1	2	5	9	FALSE	TRUE	FALSE	FALSE
1	1	2	6	10	TRUE	FALSE	TRUE	FALSE
1	1	2	7	11	FALSE	FALSE	FALSE	TRUE
1	1	2	8	12	TRUE	TRUE	FALSE	FALSE
1	1	2	9	13	FALSE	FALSE	FALSE	TRUE
1	1	3	1	6	TRUE	TRUE	FALSE	FALSE
1	1	3	2	7	FALSE	FALSE	FALSE	TRUE
1	1	3	3	8	TRUE	FALSE	FALSE	FALSE
1	1	3	4	9	FALSE	TRUE	FALSE	FALSE
1	1	3	5	10	TRUE	FALSE	TRUE	FALSE
1	1	3	6	11	FALSE	FALSE	FALSE	TRUE
1	1	3	7	12	TRUE	TRUE	FALSE	FALSE
1	1	3	8	13	FALSE	FALSE	FALSE	TRUE
1	1	3	9	14	TRUE	FALSE	FALSE	FALSE
1	1	4	1	7	FALSE	FALSE	FALSE	TRUE

Image 3.1.1 Screenshot of working carried out for Research Question 1 (i) in [Excel file](#)

(ii) We found that the probability of any 5 cards being a multiple of 2 is $\frac{33889}{67778}$, probability of the number being a multiple of 3 is $\frac{22592}{67778}$, the probability of any 5 cards being a multiple of 5 is $\frac{13550}{67778}$ and probability of any 5 cards being a prime number is $\frac{10014}{67778}$.

Number 1	Number 2	Number 3	Number 4	Number 5	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime Number?	
1	1	1	1	1	2	TRUE	TRUE	FALSE	FALSE	
1	1	1	1	1	3	FALSE	FALSE	FALSE	TRUE	
1	1	1	1	1	4	TRUE	FALSE	FALSE	FALSE	
1	1	1	1	1	5	FALSE	TRUE	FALSE	FALSE	
1	1	1	1	1	6	TRUE	FALSE	TRUE	FALSE	
1	1	1	1	1	7	FALSE	FALSE	FALSE	TRUE	
1	1	1	1	1	8	TRUE	TRUE	FALSE	FALSE	
1	1	1	1	1	9	FALSE	FALSE	FALSE	TRUE	
1	1	1	1	2	1	6	TRUE	TRUE	FALSE	FALSE
1	1	1	2	2	2	7	FALSE	FALSE	FALSE	TRUE
1	1	1	2	3	3	8	TRUE	FALSE	FALSE	FALSE
1	1	1	2	4	4	9	FALSE	TRUE	FALSE	FALSE
1	1	1	2	5	5	10	TRUE	FALSE	TRUE	FALSE
1	1	1	2	6	6	11	FALSE	FALSE	FALSE	TRUE
1	1	1	2	7	7	12	TRUE	TRUE	FALSE	FALSE
1	1	1	2	8	8	13	FALSE	FALSE	FALSE	TRUE
1	1	1	2	9	9	14	TRUE	FALSE	FALSE	FALSE
1	1	1	3	1	7	FALSE	FALSE	FALSE	TRUE	
1	1	1	3	2	8	TRUE	FALSE	FALSE	FALSE	
1	1	1	3	3	3	9	FALSE	TRUE	FALSE	FALSE
1	1	1	3	4	4	10	TRUE	FALSE	TRUE	FALSE
1	1	1	3	5	5	11	FALSE	FALSE	FALSE	TRUE
1	1	1	3	6	6	12	TRUE	TRUE	FALSE	FALSE
1	1	1	3	7	7	13	FALSE	FALSE	FALSE	TRUE
1	1	1	3	8	8	14	TRUE	FALSE	FALSE	FALSE
1	1	1	3	9	9	15	FALSE	TRUE	TRUE	FALSE
1	1	1	4	1	8	TRUE	FALSE	FALSE	FALSE	
1	1	1	4	2	9	FALSE	TRUE	FALSE	FALSE	
1	1	1	4	3	10	TRUE	FALSE	TRUE	FALSE	
1	1	1	4	4	4	11	FALSE	FALSE	FALSE	TRUE

Image 3.1.2 Screenshot of working carried out for Research Question 1 (ii) in [Excel file](#)

Sections/Probability	Multiples of 2	Multiples of 3	Multiples of 5	Prime Number
Research Question 1 (i)	$\frac{3289}{6577}$	$\frac{2201}{6577}$	$\frac{1317}{6577}$	$\frac{1860}{6577}$
Research Question 1 (ii)	$\frac{33889}{67778}$	$\frac{22592}{67778}$	$\frac{13550}{67778}$	$\frac{10014}{67778}$

Table 3.1 Table of probability obtained from Research Question 1

3.2 Research question 2

- (i) We found the probability of 4 cards without repeats being multiple of 2 is $\frac{60}{124}$, and the probability of any 4 cards being a multiple of 3 is $\frac{43}{124}$, the probability of any 4 cards being a multiple of 5 is $\frac{26}{124}$, and the probability of any 4 cards being a prime number is $\frac{37}{124}$.

Number 1	Number 2	Number 3	Number 4	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime number?
1	2	3	4	10	TRUE	FALSE	TRUE	FALSE
1	2	3	5	11	FALSE	FALSE	FALSE	TRUE
1	2	3	6	12	TRUE	TRUE	FALSE	FALSE
1	2	3	7	13	FALSE	FALSE	FALSE	TRUE
1	2	3	8	14	TRUE	FALSE	FALSE	FALSE
1	2	3	9	15	FALSE	TRUE	TRUE	FALSE
1	2	4	5	12	TRUE	TRUE	FALSE	FALSE
1	2	4	6	13	FALSE	FALSE	FALSE	TRUE
1	2	4	7	14	TRUE	FALSE	FALSE	FALSE
1	2	4	8	15	FALSE	TRUE	TRUE	FALSE
1	2	4	9	16	TRUE	FALSE	FALSE	FALSE
1	2	5	6	14	TRUE	FALSE	FALSE	FALSE
1	2	5	7	15	FALSE	TRUE	TRUE	FALSE
1	2	5	8	16	TRUE	FALSE	FALSE	FALSE
1	2	5	9	17	FALSE	FALSE	FALSE	TRUE
1	2	6	7	16	TRUE	FALSE	FALSE	FALSE
1	2	6	8	17	FALSE	FALSE	FALSE	TRUE
1	2	6	9	18	TRUE	TRUE	FALSE	FALSE
1	2	7	8	18	TRUE	TRUE	FALSE	FALSE
1	2	7	9	19	FALSE	FALSE	FALSE	TRUE
1	2	8	9	20	TRUE	FALSE	TRUE	FALSE
1	3	4	5	13	FALSE	FALSE	FALSE	TRUE
1	3	4	6	14	TRUE	FALSE	FALSE	FALSE
1	3	4	7	15	FALSE	TRUE	TRUE	FALSE
1	3	4	8	16	TRUE	FALSE	FALSE	FALSE
1	3	5	6	15	FALSE	TRUE	TRUE	FALSE
1	3	5	7	16	TRUE	FALSE	FALSE	FALSE
1	3	5	8	17	FALSE	FALSE	FALSE	TRUE
1	3	5	9	18	TRUE	TRUE	FALSE	FALSE
1	3	6	7	17	FALSE	FALSE	FALSE	TRUE

Image 3.2.1 Screenshot of working carried out for Research Question 2 (i) in [Excel file](#)

- (ii) We found that the probability of any 5 cards being a multiple of 2 is $\frac{67}{121}$, the probability of any 5 cards being a multiple of 3 is $\frac{40}{121}$, the probability of any 5 cards being a multiple of 5 is $\frac{25}{121}$, and the probability of any 5 cards being a prime number is $\frac{30}{121}$.

Number 1	Number 2	Number 3	Number 4	Number 5	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime number?
1	2	3	4	5	15	FALSE	TRUE	TRUE	FALSE
1	2	3	4	6	16	TRUE	FALSE	FALSE	FALSE
1	2	3	4	7	17	FALSE	FALSE	FALSE	TRUE
1	2	3	4	8	18	TRUE	TRUE	FALSE	FALSE
1	2	3	4	9	19	FALSE	FALSE	FALSE	TRUE
1	2	3	5	6	17	FALSE	FALSE	FALSE	TRUE
1	2	3	5	7	18	TRUE	TRUE	FALSE	FALSE
1	2	3	5	8	19	FALSE	FALSE	FALSE	TRUE
1	2	3	5	9	20	TRUE	FALSE	TRUE	FALSE
1	2	3	6	7	19	FALSE	FALSE	FALSE	TRUE
1	2	3	6	8	20	TRUE	FALSE	TRUE	FALSE
1	2	3	6	9	21	FALSE	TRUE	FALSE	FALSE
1	2	3	7	8	21	FALSE	TRUE	FALSE	FALSE
1	2	3	7	9	22	TRUE	FALSE	FALSE	FALSE
1	2	3	8	9	23	FALSE	FALSE	FALSE	TRUE
1	2	4	3	4	14	TRUE	FALSE	FALSE	FALSE
1	2	4	3	5	15	FALSE	TRUE	TRUE	FALSE
1	2	4	3	6	16	TRUE	FALSE	FALSE	FALSE
1	2	4	3	7	17	FALSE	FALSE	FALSE	TRUE
1	2	4	3	8	18	TRUE	TRUE	FALSE	FALSE
1	2	4	3	9	19	FALSE	FALSE	FALSE	TRUE
1	2	4	5	6	18	TRUE	TRUE	FALSE	FALSE
1	2	4	5	7	19	FALSE	FALSE	FALSE	TRUE
1	2	4	5	8	20	TRUE	FALSE	TRUE	FALSE
1	2	4	5	9	21	FALSE	TRUE	FALSE	FALSE
1	2	4	6	7	20	TRUE	FALSE	TRUE	FALSE
1	2	4	6	8	21	FALSE	TRUE	FALSE	FALSE
1	2	4	6	9	22	TRUE	FALSE	FALSE	FALSE
1	2	4	6	9	22	TRUE	FALSE	FALSE	FALSE

Image 3.2.2 Screenshot of working carried out for Research Question 2 (ii) in [Excel file](#)

Table 3.2 Table of probability obtained from Research Question 2

Sections/Probability	Multiples of 2	Multiples of 3	Multiples of 5	Prime number
Research Question 2 (i)	$\frac{60}{124}$	$\frac{43}{124}$	$\frac{26}{124}$	$\frac{37}{124}$
Research Question 2 (ii)	$\frac{67}{121}$	$\frac{40}{121}$	$\frac{25}{121}$	$\frac{30}{121}$

3.3 Research question 3

- (i) We found that the probability of all cards in the deck being a multiple of 2 is $\frac{14281}{28561}$, the probability of any 4 cards being a multiple of 3 is $\frac{9550}{28561}$, the probability of any 4 cards being a multiple of 5 is $\frac{5710}{28561}$, and the probability of any 4 cards being a prime number is $\frac{4995}{28561}$.

Number 1	Number 2	Number 3	Number 4	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime number?
1	1	1	1	4	TRUE	TRUE	FALSE	FALSE
1	1	1	2	5	FALSE	FALSE	TRUE	TRUE
1	1	1	3	6	TRUE	TRUE	FALSE	FALSE
1	1	1	4	7	FALSE	FALSE	FALSE	TRUE
1	1	1	5	8	TRUE	FALSE	FALSE	FALSE
1	1	1	6	9	FALSE	TRUE	FALSE	FALSE
1	1	1	7	10	TRUE	FALSE	TRUE	FALSE
1	1	1	8	11	FALSE	FALSE	FALSE	TRUE
1	1	1	9	12	TRUE	TRUE	FALSE	FALSE
1	1	1	10	13	FALSE	FALSE	FALSE	TRUE
1	1	1	11	14	TRUE	FALSE	FALSE	FALSE
1	1	1	12	15	FALSE	TRUE	TRUE	FALSE
1	1	1	13	16	TRUE	FALSE	FALSE	FALSE
1	1	2	1	5	FALSE	FALSE	TRUE	TRUE
1	1	2	2	6	TRUE	TRUE	FALSE	FALSE
1	1	2	3	7	FALSE	FALSE	FALSE	TRUE
1	1	2	4	8	TRUE	FALSE	FALSE	FALSE
1	1	2	5	9	FALSE	TRUE	FALSE	FALSE
1	1	2	6	10	TRUE	FALSE	TRUE	FALSE
1	1	2	7	11	FALSE	FALSE	FALSE	TRUE
1	1	2	8	12	TRUE	TRUE	FALSE	FALSE
1	1	2	9	13	FALSE	FALSE	FALSE	TRUE
1	1	2	10	14	TRUE	FALSE	FALSE	FALSE
1	1	2	11	15	FALSE	TRUE	TRUE	FALSE
1	1	2	12	16	TRUE	FALSE	FALSE	FALSE
1	1	2	13	17	FALSE	FALSE	FALSE	TRUE
1	1	3	1	6	TRUE	TRUE	FALSE	FALSE
1	1	3	2	7	FALSE	FALSE	FALSE	TRUE
1	1	3	3	8	TRUE	FALSE	FALSE	FALSE
1	1	3	4	9	FALSE	TRUE	FALSE	FALSE

Image 3.3.1 Screenshot of working carried out for

Research Question 3 (i) in [Excel file](#)

- (ii) We found the probability of 4 cards without repeats being multiple of 2 is $\frac{348}{682}$, and the probability of any 4 cards being a multiple of 3 is $\frac{226}{682}$, the probability of any 4 cards being a multiple of 5 is $\frac{136}{682}$, and the probability of any 4 cards being a prime number is $\frac{163}{682}$.

Number 1	Number 2	Number 3	Number 4	Sum of digits	Multiple of 2?	Multiple of 3?	Multiple of 5?	Prime number?
1	2	3	4	10	TRUE	FALSE	TRUE	FALSE
1	2	3	5	11	FALSE	FALSE	FALSE	TRUE
1	2	3	6	12	TRUE	TRUE	FALSE	FALSE
1	2	3	7	13	FALSE	FALSE	FALSE	TRUE
1	2	3	8	14	TRUE	FALSE	FALSE	FALSE
1	2	3	9	15	FALSE	TRUE	TRUE	FALSE
1	2	3	10	16	TRUE	FALSE	FALSE	FALSE
1	2	3	11	17	FALSE	FALSE	FALSE	TRUE
1	2	3	12	18	TRUE	TRUE	FALSE	FALSE
1	2	3	13	19	FALSE	FALSE	FALSE	TRUE
1	2	4	5	12	TRUE	TRUE	FALSE	FALSE
1	2	4	6	13	FALSE	FALSE	FALSE	TRUE
1	2	4	7	14	TRUE	FALSE	FALSE	FALSE
1	2	4	8	15	FALSE	TRUE	TRUE	FALSE
1	2	4	9	16	TRUE	FALSE	FALSE	FALSE
1	2	4	10	17	FALSE	FALSE	FALSE	TRUE
1	2	4	11	18	TRUE	TRUE	FALSE	FALSE
1	2	4	12	19	FALSE	FALSE	FALSE	TRUE
1	2	4	13	20	TRUE	FALSE	TRUE	FALSE
1	2	5	6	14	TRUE	FALSE	FALSE	FALSE
1	2	5	7	15	FALSE	TRUE	TRUE	FALSE
1	2	5	8	16	TRUE	FALSE	FALSE	FALSE
1	2	5	9	17	FALSE	FALSE	FALSE	TRUE
1	2	5	10	18	TRUE	TRUE	FALSE	FALSE
1	2	5	11	19	FALSE	FALSE	FALSE	TRUE
1	2	5	12	20	TRUE	FALSE	TRUE	FALSE
1	2	5	13	21	FALSE	TRUE	FALSE	FALSE
1	2	6	7	16	TRUE	FALSE	FALSE	FALSE
1	2	6	8	17	FALSE	FALSE	FALSE	TRUE
1	2	6	9	18	TRUE	TRUE	FALSE	FALSE
1	2	6	10	19	FALSE	FALSE	FALSE	TRUE
1	2	6	11	20	TRUE	FALSE	TRUE	FALSE
1	2	6	12	21	FALSE	TRUE	FALSE	FALSE
1	2	6	13	22	TRUE	FALSE	FALSE	FALSE
1	2	7	8	18	TRUE	TRUE	FALSE	FALSE
1	2	7	9	19	FALSE	FALSE	FALSE	TRUE

Image 3.3.2 Screenshot of working carried out for Research Question 3 in [Excel file](#)

Table 3.3 Table of Probability obtained for Research Question 3

Sections/Probability	Multiples of 2	Multiples of 3	Multiples of 5	Prime number
Research Question 3 (i)	$\frac{14281}{28561}$	$\frac{9550}{28561}$	$\frac{5710}{28561}$	$\frac{4995}{28561}$
Research Question 3 (ii)	$\frac{348}{682}$	$\frac{226}{682}$	$\frac{136}{682}$	$\frac{163}{682}$

4 Conclusion

Research Questions / Percentage	Multiple of 2	Multiple of 3	Multiple of 5	Prime Number
Research Question 1 (i)	50.0%	33.4%	20.0%	28.3%
Research Question 1 (ii)	50.0%	33.3%	20.0%	14.8%
Research Question 2 (i)	48.4%	34.7%	21.0%	29.8%
Research Question 2 (ii)	55.4%	33.1%	20.7%	24.8%
Research Question 3 (i)	50.0%	33.4%	20.0%	17.5%
Research Question 3 (ii)	51.0%	34.7%	20.9%	25.0%

Table 4.1 The above data table shows the data obtained and results from each research questions in terms of percentage rounded to 3 significant figures, which is the outcome of the project.

There is a general pattern observed through all the research questions.

- the percentage for the multiples of 2 is always the highest, averaging at 50.8% throughout all research questions, and remains the same generally
- the percentage for the multiples of 3 follows behind, averaging at 33.8% for all research questions
- this is then followed by the multiple of 5 which averages at about 20.4% for all research questions
- finally, we have realized that the percentage of prime numbers follows no actual pattern, where the lowest percentage is from 14.8% and highest at 29.8%, and the average being 23.4%.

To reflect, probably the most difficult part of the project is the severe lag that we experienced on our computers while opening the excel file, due to the high number of probable cases to consider for each research question.

If we had more time, we would want to extend and continue our research with the probability of obtaining multiples of 10, using the different mathematical symbols, such as: +, -, ×, ÷, between 4 or more numbers. Furthermore, we would also want to try experimenting with C++ to solve the research questions and carry out the same project.

5 References

Apostol, T.M. (1969). *A Short History of Probability*. The Chinese University of Hong Kong, Department of Mathematics.

https://www.math.cuhk.edu.hk/course_builder/2021/math3280/A%20Short%20History%20of%20Probability.pdf

Google. (n.d.). *Google Sheets function list - Docs Editors Help*. Retrieved June 11, 2021, from <https://support.google.com/docs/table/25273>

Grubbs, M. (2016, July 13). *Google Sheets 101: The Beginner's Guide to Online Spreadsheets*. Zapier. <https://zapier.com/learn/google-sheets/google-sheets-tutorial/>

Prashanth (2019, August 21). *The Role of TRUE Logical Function in Google Sheets*. Info

Shubham Jain. (2019, April 12). *How to Create a Custom Formula in Google Sheets?* <https://shubhamjain.co/til/custom-formula-google-sheet/>

Kemp, A. (2015, November 14). *How to count TRUE entries in a column [closed] - Ask LibreOffice*. LibreOffice. <https://ask.libreoffice.org/en/question/16615/how-to-count-true-entries-in-a-column/>