

SMTTP (Math) Final Evaluation
Written Report

Divide and Conquer

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1. Abstract

This Math Research Project is a study about using digital optimisation tools within a Mathematical Model to optimise the transshipment processes in Singapore's transshipment ports. In our investigation, we started off by gaining a deeper insight and understanding of the Mathematical Problem presented to us. Next, with a higher level of comprehension, we started by identifying our objective function and constraints present within the optimisation problem. Once they have been correctly and accurately identified, we started to represent and articulate our problem in Mathematical terminologies and variables before embarking on the journey of defining our terminologies and variables used vigorously. Next, we started to manually solve for the most efficient and optimal solution of a simplified model by hand, starting with only 1 variable at play before increasing the complexity of the model by introducing other variables at play and varying their values, gaining a better understanding of the interactions between these variables at the same time. Lastly, we created a digital Mathematical Model on Microsoft Excel and utilised the SOLVER optimisation tool present to automate the solving for the most efficient and optimal solution to our transshipment problem.

The purposes and aims of this project are:

1. To understand how transshipment ports and distribution processes work
2. To understand how the different variables and factors at play in transshipment functions interact with each other and how they can affect the optimal solution
3. To devise a digital and automated Mathematical Model using optimisation tools to reach the most efficient and optimal solution that can apply to various scenarios

After the research we have conducted, we have a much better understanding of how transshipment works in real life and its dynamics and intricacies that make the optimisation of transshipment processes difficult. We have also managed to come out with the Mathematical Model that can be applied to various scenarios by tweaking the values of the variables and we sincerely hope that this is indeed one step closer in mankind's pursuit towards finding the most optimal solution that can be leveraged upon in transshipment ports and processes all around the world, specifically in Singapore to reap the sheer economic benefits generated by transshipment and be a leading global transshipment port and model trading hub.

2. Introduction

2.1 Brief Introduction (Rationale)

Singapore is one of the world's largest transport hubs, with a trade value amounting to 3 times its Gross Domestic Product (GDP) as of 2017. With roughly 1,000 vessels in its seaports at any time, it is evident that Singapore's seaports are very busy ones. Furthermore, they attract 130,000 vessel calls on average in a year and this maritime industry makes up 7% of Singapore's GDP and 170,000 jobs, a figure that is projected to grow after former Minister for Manpower Josephine Teo announced that 13,000 more jobs will be established in the maritime and other industries by 2025 (Turner, 2021). Undoubtedly, Singapore's seaports are extremely vital to Singapore's economic landscape and its success, given Singapore's geostrategic location right in the heart of Asia, lying on the Malacca Straits as the gateway between the East and Western countries. Therefore, it is critical that Singapore's seaports are as optimised as possible to ensure Singapore's future economic growth, complemented by its export-oriented free-market economy. As we recognise the limitation that no transshipment system can be 100% optimised, this project aims to further optimise the functioning processes of Singapore's local seaports to ensure their continued competitiveness and to secure their global position as one of the most efficient seaports around the world. With the upcoming construction of the Tuas mega-port, Singapore's international position as a global transshipment hub is becoming ever more important, thus, this highlights the need for Singapore's seaports to be as optimised as possible. Therefore, the primary fundamental aim of this project is to devise the most efficient way to distribute Ships over Machines while factoring in other variables such as Workload (of Ships), Rate of Work, and Arrival Times that realistically come into play in real-life transshipment processes to represent, model and simulate this Mathematical Problem presented to us more accurately for real-life applications.

2.2 Basic Definitions of Fundamental Terminologies

2.2.1 Objective

Term	Explanation
Most Efficient/Optimal	Shortest possible time to fully load the Ships

2.2.2 Variables

Term	Explanation
Workload	Number of containers that need to be loaded
Machines	Cranes used to load Ships
Arrival Hour	The hour in which the Ships arrive
Rate of Work	Total number of containers loaded/hour
Operational Fixed Cost	The Fixed Cost incurred per hour whenever any of the Machines are being activated in that particular hour
Delay Cost	The Fixed Cost incurred per hour for every unloaded container in that particular hour
Productivity	100% Productivity Eff.(100% of Rate of Work) for Machines loading 1 Ship at a time; 80% Productivity Eff.(80% of Rate of Work) for Machines loading more than 1 Ship simultaneously for example (An assumption)

2.2.3 Concepts

Term	Explanation
Containers	Cargo that is loaded onto a Ship
Transshipment	Transport of goods/containers to an intermediate destination before the final one
Yard Operations	Processes that occur in a yard (e.g., storage of containers)

2.3 Research Problems

In this project, we raised a few initial Research Questions and Problems to be answered along the way, step by step and one at a time to guide us throughout the course of this project.

Our problem is defined as, how do we optimise the allocation of Machines to Ships, in order to minimise the total time needed to fully load the Ships. We have built a model that allows us to

factor in the following variables which are, the amount of cargo a Ship has or its Workload, the Rate of Work of the Machines, the Productivity of the Machines, the Arrival Hour of the Ships, the Operational Fixed Cost of the activated Machines, and the value of cargo that the Ships are carrying or the Delay Cost. As our problem focuses primarily on the allocation of Machines, it is possible for one Ship to be loaded by an infinite number of Machines, and for a Machine to load an infinite number of Ships. However, we have implemented a Productivity decrease if a Machine loads more than one Ship to make it more realistic, and we only consider the case of 2 Ships and 2 Machines, where it is more realistic to assume that a Ship can be loaded by 2 Machines, and a Machine is able to load all Ships berthed.

We have also included the following constraints, which are that the total allocation of Machines cannot exceed its Rate of Work to simulate situations in real-life, the allocation of each Machine to each Ship must be an integer value, the remaining Workload must be larger than or equal to 0 to prevent extra allocation and work of the Machines when there is no Workload to be loaded or when the Ships are already fully loaded, and at the end of the 8 hours, the remaining Workload to be processed must be 0 to ensure all the Ships are fully loaded.

The initial Research Problems and Questions posed are as follows:

1. What is the most efficient and optimal way of distributing 2 Ships over 2 Machines, given that all other variables, including the Rate of Work of the Machines, Workload of the Ships and Arrival Hour remain constant?

2. What is the most efficient and optimal way of distributing Ships with Workload, over Machines with a Rate of Work, and varying arrival times (Arrival Hour) when changed one at a time?
3. What is the most efficient and optimal way of distributing Ships over Machines and allocating the Rate of Work of Machines to Ships when there are no constant variables and all the variables are in play?

2.4 Aims of Project

1. To investigate the various possible solutions and determine which solution is the most efficient and optimal for the 1st simplified case of 2 Ships and 2 Machines
2. To understand how the 8 different variables: Number of Ships, Number of Machines, Rate of Work of Machines, Workload of Ships, Arrival Hour, Operational Fixed Cost, Delay Cost and Productivity affect the most efficient and optimal solution found in Objective 1
3. To develop a digital, automated computational Mathematical Model using Excel, its optimisation tools to determine the most efficient and optimal solution when all 8 variables come into play

2.5 Field of Mathematics

1. Generalised Reduced Gradient (GRG) (Non-Linear)

3. Literature Review

3.1 Current Optimisation of Sea Port Shipments

Currently, with the rising importance of seaports globally, there has been extensive research on the optimisation of seaport shipments, and one such problem identified, which lies in the optimisation process, is the Berth Allocation Problem with Quay Crane Assignment Problem. One such study was conducted where a method based on the rolling-horizon strategy was developed, and a mixed-integer multi-objective linear programming model was established (Xiao and Hu, 2014). By introducing penalty points, which corresponded to factors such as delayed time for berthing or departure compared to the estimated time, the model was able to devise the most optimal method using a rolling-horizon approach.

However, there is a lack of research pertaining to Singapore specifically in this field, despite it being one of the most important seaports globally, and this project investigates a wider range of variables such as the number of Ships and Workload, as compared to the above-mentioned problem which only focuses primarily on berthing, therefore, the above-mentioned problem's scope is narrower and does not completely align with our project's broader scope. Thus, this project will aim to conduct a similar study with a different set of variables based more on crane assignment.

Furthermore, in order to minimise the housekeeping costs generated by transshipment flows between ships and to maximise the total value of quay crane profiles (i.e: number of quay cranes per loading time), one study (Giallombardo et al, 2009) has used a mixed-integer quadratic program and a linearisation program which reduces to a mixed-integer linear program.

The mixed-integer or quadratic program employed by the 2 studies above refers to a mathematical optimisation program where most, or all of the variables are integers. This applies to our project as the numbers we are dealing with, be it the number of ships, the number of cranes etc are all integers. Using constraints derived from parameters, inequalities, no matter whether it is linear or quadratic can be formed and plotted onto a coordinate plane. These inequalities will form areas on the coordinate plane, and the maximum/minimum point of the area that satisfies every, or most of the inequality will give us the most optimal solution to our problem.

3.2 Automation of Sea Ports Around the World

According to top industry analysts, seaports around the world have adopted automation at a slower rate as compared to other comparable sectors (Hugh R. Morley, 2019). However, the pace of seaports' automation is starting to pick up and accelerate due to the plethora of benefits and advantages that automation provides, such as increased efficiency and optimisation of seaports. In fact, according to a survey conducted by Mckinsey, a management consultant company, with the Shanghai International Port Group, 80% of the respondents expect that in the next five years, at least half of all greenfield seaport projects will be semi- or fully automated, clearly reflecting that the automation of seaports is becoming the global trend. Despite that, the implementation difficulties and challenges are significant (Hugh R. Morley, 2019), but careful planning can surmount them and bring about greater automation of seaports across the globe.

Seaports' automation can be defined as the use of integrated technology to develop intelligent solutions for efficient control of traffic and trade flows on the port thereby increasing port capacity and port efficiency. Its 3 principal areas include the gates, the Ship-to-Shore cranes, and the stacks. It involves the use of mechanical, hydraulic, pneumatic, electric, electronic and computerised

elements or systems to control equipment and processes. It also involves for example the automation of stacking yards and horizontal transfers between the quay and the yard (Frese, 2021). The integration of technology such as the deployment of cloud-based software to assist in creating the operational flows that help the port function smoothly are all elements of seaport automation ("Everything You Need to Know About Port Automation - SHM Blog", 2021).

One such obstacle hindering the progress of automation in seaports worldwide is fuelled by the widespread fear of people working in the industry being replaced and losing their jobs. The action of implementing and adopting such automation solutions poses a huge problem as well.

Qingdao New Qianwan Container Terminal, Port of Qingdao; Yangshan Deep Water Port, Port of Shanghai; Victoria International Container Terminal, Port of Melbourne; Maasvlakte 2, Port of Rotterdam (2018) are all existing successful automated seaports models and back in Singapore, the Tuas Sea Mega-Port is set to embrace automation and become the world's largest fully automated terminal.

Automation of seaports around the world comes with numerous benefits. Automated seaports are significantly safer than conventional ones, causing the decline in human-related accidents and makes the performance more predictable. It would help to avoid seaport congestion and reduce delays, thus indirectly reducing the waiting time for Ships due to overlapping arrival times (link to this project) and hence, increasing overall efficiency. By reducing the size of the workforce and eliminating human error, automated work sequences and algorithms are much faster and achieve increased efficiency and productivity.

Port 1.0 to 2.0 to 3.0, where automated equipment and algorithms run and optimise port processes. Now, looking beyond automated machinery, reimagining the future model of seaport automation, Port 4.0 will be powered by Artificial Intelligence (A.I.), increasing seaports' optimisation through advanced analytics and dynamic scheduling such as real-time berth planning, predictive maintenance of key assets, automated yard planning and demand planning at gate etc.

Therefore, increasing automation in seaports around the world will go a long way in increasing seaports' efficiency and in ensuring that their processes are functioning at optimal levels.

The automation of seaports around the world is extremely relevant to Singapore, as can be seen in the upcoming Tuas Sea Mega-Port in Singapore, hence, this is aligned with our project rationale (as mentioned in the Introduction and Rationale Section above). The advantages that automation brings about include increasing the overall efficiency of seaports, which helps to achieve this project's objective as well. This literature review further inspired this project to integrate elements of automation into our final solution and product: the mathematical computational model (as mentioned in the Objectives Section above).

4. Methodology

Firstly, we used the simplest case of 2 Machines and 2 Ships and generated all the possible ways to allocate the Machines to those Ships. After generating each possible case, we tried to come up with an optimal solution without varying any variables. Following this, we created separate Excel sheets to investigate how varying variables such as the number of Machines, number of Ships, Rate of Work of the Machines, Workload of the Ships, and the Arrival Hour of the Ships would affect this most optimal solution. Through this, we gained a better understanding of the problem. Afterwards, we combined all the variables into one Excel sheet, developing a system of interconnected cells governed by linear equations, based on the values we have chosen to assign to the various variables, such as the Workload of a specific Ship. Lastly, we varied multiple variables at once while inputting a set of constraints in the form of linear inequalities, and our objective function, which is the total cost to be minimised to achieve our objective of minimising the total time needed to fully load the Ships as well, into the Excel solver which utilises a non-linear Generalised Reduced Gradient (GRG) method to generate the most optimised solution to allocate the Machines to fully load the Ships.

5. Working Process

5.1 First Stage—The simplest case of 2 Machines loading 2 Ships

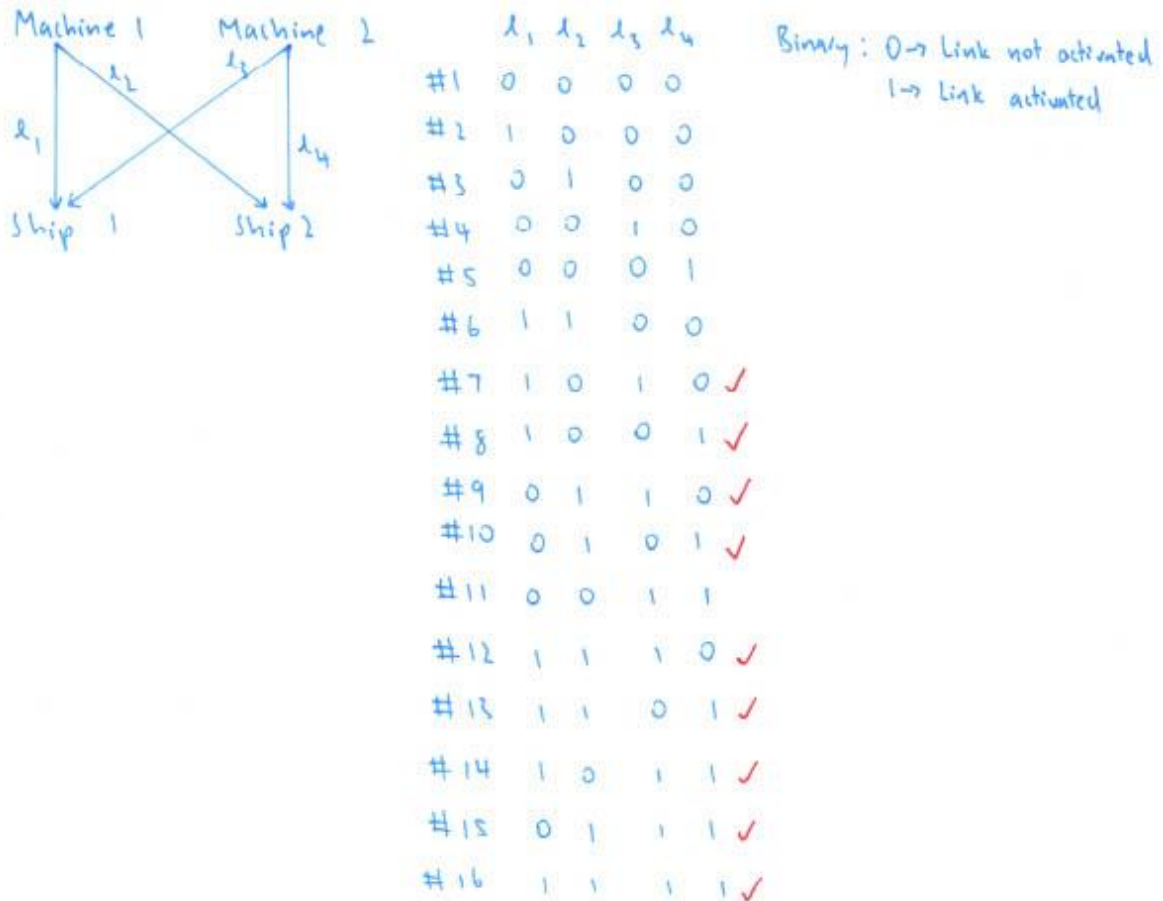


Figure 5.1

l_1 refers to a link, which refers to the allocation of one specific Machine to one specific Ship.

l_1 is activated when Machine 1 is loading Ship 1 and this activation is denoted by the binary number 1 in the table in the middle of Figure 5.1 as can be seen in the Legend at the top right-hand corner of Figure 5.1.

l_2 is activated when Machine 1 loads Ship 2 and this activation is denoted similarly to that of l_1 .

l_3 is activated when Machine 2 loads Ship 1 and this activation is denoted similarly to that of l_1 .

l_4 is activated when Machine 2 loads Ship 2 and this activation is denoted similarly to that of l_1 .

If the links are not activated, they will be denoted by the binary number 0 in the table in the middle of Figure 5.1 as can be seen in the Legend at the top right-hand corner of Figure 5.1.

The cases with ticks beside them indicate that the Ships will be fully loaded after one decision of how to allocate the Machines is made, and hence another decision is not needed to fully load both Ships.

We see each link as a possible decision, and thus the total number of possible decisions was calculated by multiplying the decision variables, namely the number of Ships and the number of Machines, together.

Therefore, the total number of possible cases = $2^{\text{total number of possible decisions}}$. The exponential has a base of 2 due to how the links are binary in nature. For example, when we have 2 Machines and 2 Ships, there will be 2×2 possible decisions. As a result, there will be $2^4 = 16$ number of cases.

5.2 Second Stage—Varying variables such as number of Machines, number of Ships, Workload, Rate of Machines and Arrival Times individually, and manually optimising each possible case

Firstly, we decided on a specific variable that we wanted to investigate. We would increase that variable linearly (E.g.: Number of Ships from 1 to 2 to 3 etc) while keeping all other variables constant. Through manually calculating the most optimal way to load the Ships for each case in 10 iterations of that particular variable, we then formed a general formula to calculate the total berthing time to load the Ship(s) for n iterations of the variable, seeking to minimise the berthing time required to load all the Ships.

Generally, we have found that it is more optimal to load one Ship at a time while letting the other Ships wait instead of loading the Ships simultaneously. In the case of varying Workloads, it is more optimal to load the Ships in ascending order of their Workload, while at the same time continuing to load the Ships one at a time. In the case of varied Arrival Times, it is more optimal to load the Ship which comes first, while still continuing to load the Ships one at a time.

This Second Stage, while not being exactly useful in telling us the most optimal way to load the Ships while varying the various variables one at a time, allowed us to understand our problem in greater detail, and hence, allowing us to be able to better define our variables, our objective function and constraints for the Third Stage which we seek to devise the most optimal solution so that we would be able to create a more rigorous definition of our problem in preparation for the Third Stage.

5.3 Third Stage—Using Excel to generate the most optimal way to load the Ships

After gaining a deeper insight into our problem and coming up with the rigorous definitions for our variables, our objective function and constraints in the Second Stage above, we represented them mathematically in the form of interconnected linear equations and inequalities into a digital mathematic tool, Excel, to automate the solving for the most optimal way to load the Ships given more complex scenarios when we vary all the variables simultaneously. We utilised the mathematical optimisation tool known as Excel solver and used the solver method, non-linear Generalised Reduced Gradient (GRG) (will be elaborated upon in 7. Extension) to assist us in finding the most optimal solution to load the Ships.

5.3.1 Allocation

Allocation		Hour	1	2	3	4	5	6	7	8
Machine 1	Ship 1	20	18	8	7	7	6	6	6	6
	Ship 2	0	0	0	11	11	13	13	13	13
Machine 2	Ship 1	8	9	9	0	0	0	0	0	3
	Ship 2	0	0	0	10	8	10	10	10	4

Figure 5.3.1.1

Firstly, we have the table on Allocation, where each cell represents the Rate of Work of the Machines assigned to a particular Ship in a particular hour. We have limited ourselves to 2 Machines and 2 Ships for a duration of 8 hours to simplify things. However, should we have more Ships and/or Machines that would require a longer duration, this digital mathematical model can always be extended manually to include more rows and columns. The highlighted cells are our variable cells i.e., the values in these cells are meant to be varied and changed to devise the most optimal solution. Unfortunately, the number of Ships, Machines and number of hours required to load the Ships cannot be controlled by simply setting them to be a certain number. This is due to limitations in Excel, where the computer cannot create a new cell for more Machines, Ships or hours needed to load the Ships. Increasing the workload in turn may cause the solver to be unable

to find a solution due to 8 hours not being enough to load the Machines. However, such a problem can be solved by complementing Excel with another computing/programming software such as Python to dynamically create cells for more Machines, Ships or hours required to load the Ships. (Will be elaborated in 8. Future Research)

Total Allocation (Machines)										
Machine 1		20	20	20	20	20	20	20	0	0
Machine 2		10	10	10	10	10	10	10	10	0

Figure 5.3.1.2

Next, we have the table on Total Allocation (Machines), defined to be the total Rate of Work of the Machines being utilised. For example, the Total Allocation of Machine 1 = Machine 1’s Allocation to Ship 1 + Machine 1’s Allocation to Ship 2. This applies throughout the Machines.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.1.3

The table we have in Figure 5.3.1.3 is the table of our Variables, namely the Rate of Work of Machines, the Workload of Ships, the Arrival Hour, and new additions such as Operational Fixed Cost and Delay Cost. We are able to vary the values under these Variables to model after different scenarios and find the respective optimal solutions for them. In this sub-section, we will focus on the highlighted portion on the Rate of Work of the Machines. The values under this variable are the Rate of Work of Machines 1 and 2 respectively. This would be the first constraint in our mathematical model, such that the Total Allocation of Machines 1 and 2 (in Figure 5.3.1.2) \leq The Rate of Work of Machines 1 and 2 respectively (in Figure 5.3.1.3).

Total Allocation Eff. (Machines)									
Machine 1		20	20	20	20	20	20	0	0
Machine 2		10	10	10	10	10	10	10	0

Figure 5.3.1.4

Additionally, we have the table on Total Effective Allocation (Machines) defined to be the effective total Rate of Work of the Machines assigned to the Ship(s). In real-life, when the Machines are assigned to load more than one Ship simultaneously, the time taken to travel between the Ships to load them coupled with the natural reduction in the Rate of Work of the Machines for each Ship would result in an overall productivity decrease of the Machines. We intend to factor that into our calculations.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.1.5

Therefore, in the same table of Variables, we have another Variable, Productivity, in the highlighted portion. For example, if the productivity decrease is taken to be 20%, the value assigned under this variable should be 0.8. The Effective Allocation is then calculated in such a way: For example, if both Machine 1’s Allocation to Ship 1 and Ship 2 > 0 (using if-function with conditions in Excel), this implies that Machine 1 is loading Ship 1 and 2 simultaneously which would lead to a productivity decrease. Hence, the Effective Allocation of Machine 1 = Total Allocation of Machine 1 x Productivity. However, if both Machine 1’s Allocation to Ship 1 and Ship 2 is not > 0, this means that Machine 1 is not loading both Ships simultaneously and hence, there is no Productivity decrease, therefore, the Effective Allocation of Machine 1 = Machine 1’s Total Allocation.

Allocation Eff. (Containers/h) (Ships)									
Machine 1	Ship 1	20	20	20	0	0	0	0	0
Machine 2		10	10	10	10	0	0	0	0
Machine 1	Ship 2	0	0	0	20	20	20	0	0
Machine 2		0	0	0	0	10	10	10	0

Figure 5.3.1.6

For this section, we have the tables on Effective Allocation (Containers/h) (Ships) for Ships 1 and 2 respectively, shown in Figure 5.3.1.6, and defined to be the effective allocation of Machine 1 and 2 to Ships 1 and 2 respectively. This is how Effective Allocation (Containers/h) (Ships) is computed: Similar to how the Effective Allocation is computed— if the Effective Allocation of Machine 1 < Total Allocation of Machine 1 (using if-function with conditions in Excel), this suggests that there is a Productivity decrease, which further means that Machine 1 loaded Ships 1 and 2 simultaneously. Hence, the Effective Allocation (Containers/h) (Ships) of Machine 1 to Ship 1 = Allocation of Machine 1 to Ship 1 x Productivity. On the contrary, if the Effective Allocation of Machine 1 is not < Total Allocation of Machine 1, this means that Machine 1 is only loading 1

Ship at once and hence there is no Productivity decrease and therefore, the Effective Allocation of Machine 1 = Total Allocation of Machine 1.

Sum of Allocation Eff. (Ships)										
Ship 1		30	30	30	10	0	0	0	0	0
Ship 2		0	0	0	20	30	30	10	0	0

Figure 5.3.1.7

Next, we have the Sum of Effective Allocation for Ships 1 and 2. This can be shown in Figure 5.3.1.7 For example, the total Effective Allocation for Ship 1 = Effective Allocation of Ship 1 by Machine 1 + Effective Allocation of Ship 1 by Machine 2. Similarly, the total Effective Allocation for Ship 2 = Effective Allocation of Ship 2 by Machine 1 + Effective Allocation of Ship 2 by Machine 2. The Sum of Effective Allocation is useful because it allows us to easily find the remaining workload by subtracting the Workload arrived at a particular hour by Effective Allocation at that particular hour throughout the entire 8 hours duration. (Explained in greater detail in 5.3.2 Workload)

5.3.2 Workload

Workload arrived									
	Hour	1	2	3	4	5	6	7	8
Ship 1		100	0	0	0	0	0	0	0
Ship 2		0	0	0	90	0	0	0	0

Figure 5.3.2.1

Next, we have a table on the Workload Arrival, defined to be the total Workload of the Ships arriving at a particular hour. For example, Figure 5.3.2.1 shows that at the first hour, a total Workload of 100 of Ship 1 arrived to be loaded followed by a total Workload of 90 of Ship 2 arriving at the fourth hour to be loaded, with no Workloads from both Ship 1 and 2 arriving at other hours. This means that Ship 1 arrived in the first hour, and it has a Workload of 100 while Ship 2 arrived at the fourth hour, and it has a Workload of 90.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.2.2

The highlighted portion in the same table of Variables reflects our Arrival Hour variable. As can be seen for example, when the Arrival Hour for Ship 1 is the first hour, Ship 1's total Workload of 100 arrives at the first hour to be loaded, likewise for Ship 2 in Figure 5.3.2.1.

Remaining Workload									
Ship 1	0	72	45	28	22.4	16.8	12	7.2	0
Ship 2	0	0	0	0	71.2	54.4	34	13.6	0

Figure 5.3.2.3

Next, we have the table on the Remaining Workload, defined to be the remaining Workload of the Ships left to be loaded at each hour after whatever loading (if any) of the Ships had occurred during that hour. The Remaining Workload of Ship 1 for example is calculated in this way: the Total Workload of Ship 1 arrived at the particular hour – Sum of Effective Allocation of Ship 1 at that particular hour. However, to prevent the Machines from allocating when no Workload has arrived and from doing extra work to prevent additional unnecessary Operational Cost to be incurred, another constraint would be imposed on the Remaining Workload such that the Remaining Workload of the Ships ≥ 0 . Furthermore, to ensure that all the Ships are fully loaded by the end of the 8 hours duration, an additional constraint is added such that the Remaining Workload of the Ships at the last hour = 0.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.2.4

The highlighted portion of the same table of Variables reflects the variable, Workload of the Ships, which we can vary to model different scenarios. The Workload mentioned above refers to this variable in this Figure 5.3.2.4.

5.3.3 Costs

Operational cost										Total operational cost	Sum of Total Operational Cost	
Machine 1	150	150	150	150	150	150	150	150	150	150	1200	2800
Machine 2	200	200	200	200	200	200	200	200	200	200	1600	

Figure 5.3.3.1

Firstly, we have the table for Operational Cost, defined to be the costs incurred as long as a Machine is being activated in a particular hour.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.3.2

The Operational Fixed Cost goes as a fixed cost as can be seen in the same table of Variables under the variable Operational Fixed Cost and these fixed costs can be varied as well. The Operational

Fixed Costs of the Machine are computed as follows: For example, if the Total Allocation of Machine 1 at a particular hour > 0 , this can be inferred that at that particular hour, Machine 1 is being activated, hence, it will incur the Operational Fixed Costs for itself at that particular hour. On the other hand, if the Total Allocation of Machine 1 at a particular hour $= 0$, this means that Machine 1 is not being activated, thus, it will not incur any Operational Fixed Costs during that particular hour. The Total Operational Cost in Figure 5.3.3.1 is the sum of the Operational Costs incurred throughout the entire 8 hours duration for Machines 1 and 2 respectively. The Sum of Total Operational Cost is the sum of the Total Operational Cost for Machines 1 and 2. Since our objective is to minimise the amount of time to fully load the Ships, it is not possible to represent it mathematically as a variable to be minimised by the Excel solver, hence, we decided to express it in the form of the Sum of Total Operational Cost which can be represented mathematically as a variable to be minimised by the Excel solver. We seek to minimise the Sum of Total Operational Cost as that would indirectly minimise the amount of time taken to fully load the Ships, seeing that the Operational Cost is linearly related to the number of hours the Machines are being activated. By minimising the Sum of Total Operational Cost, the number of hours the Machines are being activated for the Ships to be fully loaded would be reduced, thereby achieving our objective to minimise the amount of time taken to load the Ships. Furthermore, the model would also seek to avoid the Productivity decrease and ensure that when the Machine is being activated, it is loading as many containers as possible to minimise the Operational Cost/container to be as economically efficient as possible. However, whether minimising the Sum of Total Operational Cost would achieve our rationale behind it can be a possibility for Future Research (will be elaborated upon in 8. Future Research).

Delay cost										Total delay cost	Sum of Total Delay Cost
Ship 1	720	450	280	224	168	120	72	0		2034	
Ship 2	0	0	0	569.6	435.2	272	108.8	0		1385.6	3419.6

Figure 5.3.3.3

Next, the table in Figure 5.3.3.3 represents the Delay Cost, defined to be the costs incurred for the delay in loading the Ships.

	Rate of Work (boxes/hr)	Operational Fixed Cost (\$/hr)	
Machine 1	20	150	
Machine 2	10	200	
	Workload	Arrival Hour	Delay Cost (\$/box/hr)
Ship 1	100	1	10
Ship 2	90	4	8
Productivity	0.8		

Figure 5.3.3.4

The highlighted portion in the same table of Variables represents the Variable, Delay Cost, which can be varied as well. The Delay Cost per box per hour of the Ships is computed in this way: Delay Cost of Ship 1 (for example) x Remaining Workload of Ship 1 at a particular hour. The Total Delay Cost in Figure 5.3.3.4 is the sum of the Delay Cost over the entire 8 hours duration for Ships 1 and 2 respectively. The Sum of Total Delay Cost is the sum of the Total Delay Cost for Ships 1 and 2. Similar to the rationale behind the Sum of Total Operational Cost above, the Sum of Total Delay Cost is another mathematical representation as a variable that can be minimised by the Excel solver to attain our objective. We believe that by seeking to minimise the Sum of Total Delay Cost, since it is dependent on the Remaining Workload of the Ships, it would minimise the amount of time taken to fully load the Ships (our goal) as the mathematical model would attempt to load the Ships as fast as possible to minimise the Remaining Workload to incur fewer Delay Costs. Additionally, the ability to vary the Delay Cost allows us to model certain real-life situations when for example the goods of a particular Ship are more valuable than that of the other Ship, hence, the

Delay Cost for the Ship with goods of higher value would logically be higher as opposed to the Ship with less valuable goods. Therefore, the most optimal solution to load the Ships when there is a varying Delay Cost is an additional dimension that is factored into our mathematical model as well, to portray real-life scenarios as accurately as possible.

5.3.4 Objective Function

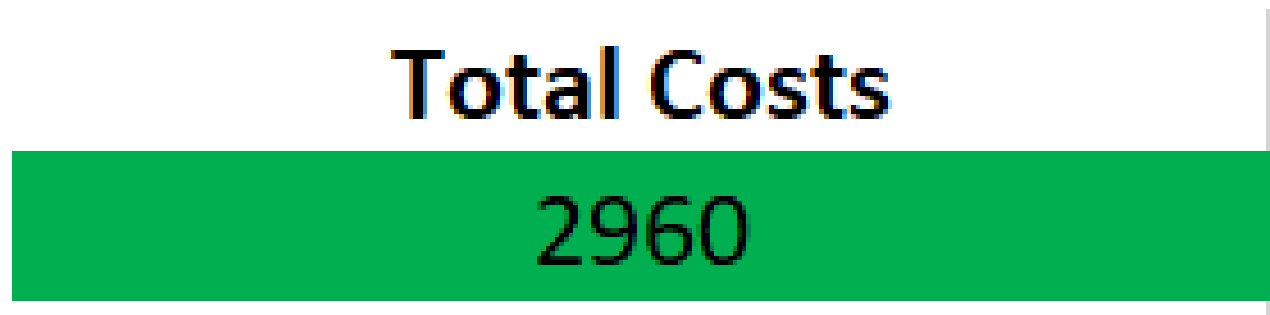


Figure 5.3.4.1

Figure 5.3.4.1 shows the Objective Function, defined to be Sum of Total Delay Cost + Sum of Total Operational Cost. When we minimise the Objective Function, the Total Operational and Delay Costs are minimised by the solver as well to achieve our objective of devising the most optimal solution to fully load the Ships, which is to load them in the shortest possible time.

5.3.5 Mathematical equations and inequalities in the Excel sheet

$(Function)_{(Machine),(Ship),(Hour)}$ will be the way in which the values for functions of a certain Machine, Ship and hour be represented. For example, let the Allocation of loading Ship 1 by Machine 1 in Hour 1 be equals to 10 containers. This will be represented as $Allocation_{1,1,1} = 10$. On the other hand, the sum of the Allocation of loading Ship 1 and Ship 2 by Machine 1 in Hour 1 will be represented as $\sum Allocation_{1,-,1}$.

Firstly, the values of $Allocation_{1,1,1}, Allocation_{1,1,2}, \dots, Allocation_{1,1,8}$, $Allocation_{2,1,1}, Allocation_{2,1,2}, \dots, Allocation_{2,1,8}$, $Allocation_{1,2,1}, Allocation_{1,2,2}, \dots, Allocation_{1,2,8}$ and $Allocation_{2,2,1}, Allocation_{2,2,2}, \dots, Allocation_{2,2,8}$ are all determined by the Solver. They are the number of containers that a certain Machine loads into a certain Ship at a certain hour.

Next, we take the sum of the Allocations, where we let Machines be the header. Total Allocations are calculated like that: $\sum Allocation_{m,-,h} = \sum_{s=1}^2 Allocation_{m,s,h}$, where $1 \leq m \leq 2$, $1 \leq h \leq 8$ and $m, h \in \mathbb{Z}$. We have placed a constraint where $\sum Allocation_{m,-,h} \leq \text{Maximum rate of work of machine } m$. This is to ensure that the constraint of the maximum Rate of Work is abided by so that Machines do not have infinite Rate of Work and thus be able to load the Ships in no time.

Next, we introduce Effective Allocation, which explained previously, accounts for a percentage decrease in the Allocation if a Machine is loading 2 Ships at once. We will firstly calculate Effective Allocation individually (Machine 1 to Ship 1, Machine 2 to Ship 1...). Effective Allocation can be calculated using an if-function where if $Allocation_{m,1,h} > 0$ and $Allocation_{m,2,h} > 0$, then $Allocation\ Eff_{m,s,h} = \lfloor Allocation_{m,s,h} \times Productivity \rfloor$, where $1 \leq m \leq 2$, $1 \leq s \leq 2$, $1 \leq h \leq 8$ and $m, s, h \in \mathbb{Z}$. Else, the Allocation values will not change. As explained previously, the productivity when a Machine is loading more than one Ship at once is a positive value less than 1. We have added the floor function as the number of containers loaded in an hour must be an integer. Previously, we have constrained the allocation to be an integer already. However, if we were to constrain the effective allocation to be an integer as well, then it

will be very difficult for the solver to find a viable solution as for example, the productivity decrease is 20%, so the productivity will be 0.8. In order for both allocation and effective allocation to be an integer, the value of allocation must have a factor of 5, which might not be the case for the most effective way to load the ship.

The Sum of Effective Allocation is calculated similarly to how the sum of the Allocations is calculated. In this case, both machines and ships can be the header. The Sum of Effective Allocation for machines is calculated as shown:

$\sum Allocation\ Eff.(Machines)_{-,s,h} = \sum_{m=1}^2 Allocation\ Eff_{.m,s,h}$, where $1 \leq s \leq 2$, $1 \leq h \leq 8$ and $s, h \in \mathbb{Z}$. The Sum of Effective Allocation for ships, is similar, calculated as: $\sum Allocation\ Eff.(Ships)_{m,-,h} = \sum_{s=1}^2 Allocation\ Eff_{.m,s,h}$, subjected to the same constraints of $1 \leq m \leq 2$, $1 \leq h \leq 8$ and $m, h \in \mathbb{Z}$.

Now, the Workload Arrived. The Workload Arrived gives information about when the Ship arrives and how much Workload it has. Both the Workload of the individual Ships and when the Ship arrives are all initial settings that we can vary. The function of Workload per ship at the hour which the Ship arrives will be the Workload of the Ship. At other hours, the workload arrived would be 0.

Now, on to the Remaining Workload. Since we are considering Workload, the header is the Ships and Machines are not considered. The Remaining workload is calculated as $Remaining\ Workload_{-,s,h} = Workload_{-,s,-} - \sum_t^h Allocation\ Eff.(Ships)_{-,s,t}$, where $1 \leq s \leq 2$, $1 \leq h \leq 8$ and $s, h \in \mathbb{Z}$. We have also added a constraint where

$Remaining\ workload_{-s,h} = 0$, if $h < Arrival\ time_{-s,-}$. This is to ensure that the Machine cannot start working on the workload that has not arrived yet. We have also added another constraint that $Remaining\ Workload_{-s,8} = 0$. This is to ensure that the remaining workload after the eight hour is 0, meaning that the Ships have been fully loaded. Lastly, we also added a constraint where $Remaining\ Workload \geq 0$, so as to ensure that the machine does not waste time doing extra work.

Next, we have the Operational Cost, which is a cost incurred when a Machine is used. Operational Cost is focussed on Machines and hence the individual Ships are not considered since we are taking the sum of the Allocation of each Ship per Machine. Operational Cost relies on an if-statement where if $\sum Allocation_{m,-,h} > 0$, then $Operational\ Cost_{m,-,h} = Operational\ Cost_{m,-,-}$, which is defined in the initial settings that we can vary as well. Else, $Operational\ Cost_{m,-,h} = 0$. These are under the constraints of $1 \leq m \leq 2$, $1 \leq h \leq 8$ and $m, h \in \mathbb{Z}$. The Operational Cost is mainly to ensure that there is no productivity decrease, as a productivity decrease will result in an increase in the cost to load one container, seeing that the fixed operational cost will still be incurred despite only being able to load a fewer number of containers per hour.

Next, we have Delay Cost, which is a cost incurred for not loading the remaining Workload onto a Ship after every hour. Delay Cost, unlike Operational Fixed Cost, is focussed on Ships instead. Only the Remaining Workload is considered, and it is independent of the Machine as it is a function of a summation of the Effective Allocation of both Machines subtracted from the Workload of a specific Ship. It is calculated as such: $Delay\ cost_{-s,h} = Remaining\ Workload_{-s,h} \times Delay\ Cost_{-s,-}$, where $1 \leq s \leq 2$, $1 \leq h \leq 8$ and $s, h \in \mathbb{Z}$. The Delay Cost is mainly to ensure

that the Ships are loaded as fast as possible for the Remaining Workload to be minimal, so as to minimise the Delay Cost.

Finally, we have the Total Cost, which is also our objective function. It is calculated as

$$Total\ cost = \sum_{h=1}^8 Operational\ Cost_{1,-h} + Operational\ Cost_{2,-h} + Delay\ Cost_{-1,h} +$$

$$Delay\ Cost_{-2,h} = \sum_{i=1}^2 \sum_{h=1}^8 Operational\ Cost_{i,-h} + Delay\ Cost_{-i,h}. \text{ This is a summation of}$$

Delay Costs and Operational Costs for every hour, and this is what we aim to minimise in this project to achieve our objective and obtain the most optimal solution to fully load the Ships in the shortest possible time given different scenarios.

6. Conclusion

Through this project, we observed a general trend throughout: At any specific time, if there is one Ship, the Machines should be allocated to loading that Ship. The Machines should always load one Ship at a time starting with the Ship with the least Workload.

When trying to find an optimal solution, there is no ideal approach that can be applied to all cases to find the optimal solution manually, without the help of the solver, especially when multiple variables come into play, as there are some cases which are exceptions, such as when one Ship has a smaller Workload than the other, however, the other Ship with a larger Workload has more valuable cargo. In such cases, usage of the solver is required to determine what would be a more efficient allocation of the Machines.

7. Extension

The Reduced Gradient (RG) method is used to find the minimum or maximum of a function subject to linear constraints, which can be inequalities or equalities. The Generalised Reduced Gradient (GRG) is an extension of the RG method to accommodate nonlinear inequality constraints. (Landon et al., 1974) In our project, our constraints are all linear and hence this section will be focussing on the RG method although we still utilised the GRG method in our Excel SOLVER. The RG method requires that the function to be optimised has to be smooth, meaning that it can be differentiated at every point on the graph, and hence allowing a gradient to be found for every point. Assume the function to be of a form $F(v_1, v_2 \dots v_n)$, where there are n variables, and there are constraints of the form $c_i(v_1, v_2 \dots v_n) = 0$ where $i = 1, \dots, m$, and also in the form $l_j \leq c_j(v_1, v_2 \dots v_n) \leq u_j$, where $j = 1, \dots, n$. The goal for the solver is to find the minimum point of the function, since in our project we want to minimise the objective function, which is the total cost. The inequalities of the second form can be transformed into something similar to the first form by introducing a slack variable s . This would cause them to be a form of $c_j(v_1, v_2 \dots v_n) + s_j = 0$. By taking cases, the inequality would be transformed into an equation. The Excel SOLVER uses a base method of gradient descent. It firstly guesses a point to start with, then finds a direction to move in to find the minimum value of the function. ∇F tells the SOLVER the gradient of the point; the direction of steepest increase. The negative of the gradient is the direction in which the solver should move to decrease the function most quickly. The function repeatedly finds a column

vector $\begin{bmatrix} \frac{\partial F}{\partial v_1} \\ \vdots \\ \frac{\partial F}{\partial v_n} \end{bmatrix}$ for n number of variables, and moves in that direction, until the column vector equals

0, meaning that the minimum of the function has been found. However, one limitation of the

SOLVER using such a quick method is that it may find a local minimum instead of a global one as both cases will cause $\nabla F = 0$, resulting in the solution found to not necessarily be the most optimal one.

8. Future Research

Since we recognise that there are limitations even within the optimisation tools themselves, and recognise the need for trade-offs, such as between speed and accuracy, we recognise that the aim of this project should be to attempt to balance and maximise both. Hence, for our future research, it is possible to look into other optimization tools such as Genetic Algorithm, that are as efficient as possible and not restricted by the limitation of the GRG method (as mentioned in 7. Extension) to devise our most optimal solution.

Furthermore, we can look at other computing programs which are better equipped to deal with cases that have multiple Ships or Machines beyond our case of merely 2 Ships and Machines. Due to the nature of Excel, it is challenging to include more Machines and Ships as the relationship between cells are based on formulas dependent on the number of cells. Furthermore, if we were to extend the model, it would simply be tedious manual work, therefore, we can consider looking at other programming software to automate such a manual process instead in our future research. Thus, we could attempt to look into incorporating the sheet with other programming software such as Python in order to enable solving cases with multiple/more Ships and Machines in order to investigate the effect of these two variables on our optimal solutions.

Additionally, we could look into accounting for where the Ships are being berthed and hence optimising the berthing of Ships, in order to make the problem more realistic. In real life, when Ships are berthed further away from a Machine, there would be an increased time required for the Machine to load that Ship and a productivity decrease.

Lastly, we could also investigate other variables, such as the Range of the Machines, and the Physical Size of the Ships, as these factors affect which Ships the Machines can load, and how many Machines the Ship can be loaded by at any point in time. We could also investigate in more detail how our chosen objective function would affect our optimal solution and if they really achieve our objectives, especially the Total Operational Cost that makes up part of our objective function.

Through the inclusion of these other variables, the changing of our chosen software to solve this problem, and a more in-depth level of investigation, we would be able to create a more realistic mathematical model, which could provide more optimal solutions that are realistically accurate and applicable in real-life, that accounts for a wider range of scenarios and can be validated by us. Although there was also the lack of time to collect data to form the base-case for a longer-term project for this Mathematical Model to be as accurate as possible coupled with the various limitations above, this project can still serve as a model or prototype which can be applied in real-life and bettered in the future through future research.

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