Mancala

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Terminology

Term Used	Description
Seed	Small objects that the players move around the board following certain rules which will be outlined below
Store	The two large depressions on the furthermost left and right of the board, used to store seeds
Pocket	The twelve smaller depressions in between the stores (mentioned above), used to store seeds
Activating	The act of selecting a pocket for that turn
Sowing	The act of placing a single seed into a pocket
Capturing	The act of taking seeds from a pocket and depositing them into the store

Introduction To Mancala

Mancala is a game originating in Africa around 3600 years ago. In this project, we used the most commonly played variation of the game, the rules of which are explained below.

The game is played with a single board which has two stores on the furthermost left and right of the board, with twelve smaller pockets in between the two stores. A total of 48 seeds is also required to play the game.

The sequence for the setup of the game before beginning gameplay is as follows:

1. Lay the board between the two players so that the rows of pockets are in front of the players and the stores are on the left and right of the players.
2. Place 4 seeds in each pocket.
3. Decide who goes first. Traditionally, the older player goes first.

After setting up the game, the rules are as follows:

1. Every turn, the player must activate a pocket. The player then sows a seed into each pocket going counterclockwise, excluding the activated pocket.
2. The store to the right of the player is that player's store. Players sow a seed in their store when they pass by them and proceed to sow seeds in the next row if they have any seeds remaining. However, players do **not** sow any seeds in the opponent's store. If they do, the seed immediately belongs to the opponent, and a take-back is not allowed.
3. If the last seed that a player sows is sowed in a store, the player in question gets another turn. This can continue indefinitely, until the player's last seed is not sown in a store.

4. If the last seed that a player sows is sowed in an **empty** pocket opposite to a pocket that has seeds in it, that player takes all the seeds in the opposite pocket and the last sown seed, and places them in his/her store. If a mistake is made and the player places the seeds in the opponent's store, the seeds immediately belong to the opponent, and a take-back is not allowed.

Notation

The type of notation we used was number-based, with some extra symbols to show extra turns and capturing.

Here is a Mancala game board which has been marked out with our numbering system. The stores are not labelled as there is no need to label them for taking down the games.

	26	25	24	23	22	21	
	11	12	13	14	15	16	

To show an extra turn, a comma is used and the next move is put after the comma as follows in the example below, which shows an opening by Player 1 utilising an extra move provided by 13.

Move Number	**Player 1**	**Player 2**
1	13, 16	...

To show a capture, the word Cap is used in brackets with the pocket seeds being captured from placed after the word as follows below, in which Player 1 captures seeds from Player 2 after Player 2 does not react defensively to the Funnel Opening, which is mentioned later.

Move Number	**Player 1**	**Player 2**
2	12 (Cap21)	...

Research Questions Posed

1. What strategies always result in a win? If there are none, what strategies give the highest chance of winning?
2. What is the longest possible game of Mancala?
3. How can we make the game more complex and fun to play? After the modifications, how have the winning strategies from the original game changed?

Relevant Field of Mathematics

One relevant field of mathematics is probability, which we used by testing games of Mancala in a simulator that we created. The simulator ran games with each player playing random moves or through utilizing various different strategies, which allowed us to see the results after we inserted the results in an Excel Spreadsheet. We also used some logic to create moves which gave the best chance of winning.

Literature Review

An article by Seth Brown (2020) details various strategies on how to win a game of mancala. The main strategies shown are taking free turn opportunities, capturing and defending against captures by looking at where your opponent's seeds would land. This is relevant to our project since these are also the main strategies we are encompassing into our simulations in Scratch, a table of which can be found in Research Question 1.

In terms of notation, there are a few types of notations used by writers. The most common formal form of notation matches notation used by Oware, as shown below:

	F	E	D	C	B	A	
-							-
	a	b	c	d	e	f	

This notation system is used in the Mancala "Fandom" website, which is one of the best sources for information on Mancala. However, we did not actually use this website for our research. This notation system is commonly used in formal tournaments, and includes the total number of seeds captured that turn in parenthesis behind the moves. To show extra moves, a hyphen (-) is used.

We did not choose this system because:

1. We did not need to display the total number of seeds as we were already showing seeds captured. We also took it that a single seed would be captured if there was a free turn.
2. We found it easier for our Scratch program to generate number sequences as compared to alphabetical sequences.

One type of notation we did consider was this one, which is used by computer programmers:

	13	12	11	10	9	8	
14							7
	1	2	3	4	5	6	

We did not use it because we found it very difficult to differentiate between the moves by Player 1 and Player 2 in our number strings.

Working Process:

The main method used here is data analysis through simulating games using a Scratch programme that ran games of Mancala in which players played random moves each turn until one player eventually won. All the simulations done are in the google drive (https://drive.google.com/drive/folders/1iv4QVA3qiaD2Yk5wU5yGGWAEKFQ3w5L-?usp=sharing). By simulating a large number of games, we were able to infer from the results what would happen given a specific strategy.

The simulations are logged in excel files and can be read somewhat easily. Take for example, this short game logged as 13614211314222413251426140 5. The 1st number shows which player won (1 for P1 won, 2 for P2 won, 0 for draw). The 2nd and 3rd numbers show the number of seeds P1 had at the end of the game, which in this case is 36. The subsequent numbers are all the moves the players did based on the notation shown above, apart from the final 2 numbers, which depict the number of rounds in the game. The strategies used are also detailed by the file name, with 1316 referring to the funnel opening for player 1 and 2928 referring to the response to the funnel opening by player 2.

Research Question 1:

What strategies always result in a win? If there are none, what strategies give the highest chance of winning?

No strategies result in a direct win for Player 1 or Player 2 in the game. There are too many uncertainties and choices for such a game to occur. So, we started to look for strategies which result in the highest win rate possible for Player 1 against Player 2 which does random moves. In order for us to find a winning strategy, we started by simulating 20,000 random games so that we can see what are players 1 and 2's win rates as a control. This shows that Player 1 has a 41.4% win rate while Player 2 has a 45.8% win rate, with the rest of the games being draws.

After this, we experimented with various methods so that we could make Player 1's win rate as high as we possibly could. This included creating an opening ourselves, which would be referred to as the funnel opening.

This was an opening created by us using our knowledge of Mancala; the notation of the opening is as follows:

Move Number	**Player 1**	**Player 2**
1	13, 16	...

This is how the board appears after this opening:

	4	4	5	5	5	5	
0							2
	4	4	0	5	5	0	

This opening moves:

Move Number	**Player 1**	**Player 2**
1	13, 16	22, 21

Player 2 plays this in order to avoid Player 1 getting too big of a headstart with a capture using:

Move Number	**Player 1**	**Player 2**
2	11/12, *depending on what Player 2 does* (Cap21)	...

Where Player 1 starts with a head start of 7-1.

In addition to forcing moves from Player 2, Player 1 can also get a free move every turn if the Funnel Opening is played correctly. Playing 16 gives Player 1 a "funnel" that can give them a free turn as well as the potential of storing at least 1 seed every turn after turn 2. In turn 2, Player 1's main objectives are as follows:

1. Punish Player 2 if they do not react defensively to 13, 16.
2. Set up the funnel by playing a move such that a seed lands in the funnel (Disregarding Player 2's move, the possible moves are as follows: 12, 14, 15.)

This allows Player 1 to play 16 every turn for an extra move and refills the funnel with whatever move they play next, as long as it is able to reach pocket 16. This may not be the best move on every turn. However, as players gain experience playing the game, they will be able to tell when to refill the funnel and when to play another move. Additionally, there isn't much Player 2 can do to sabotage the funnel, as they will have to commit too many resources to pool seeds to reach from their side of the board to pocket 16. This opens up an opportunity for Player 1 to set up a capture while Player 2 is still pooling seeds.

This opening is a very effective early game move which is proven based on our simulations where Player 1 does these opening and random moves. In said simulations, Player 1 started winning 48.5% of the time, 7% more than previously. However, this is not the only thing we can do to increase Player 1's win rate. There are two major mechanics in the game which can be used to gain more seeds. Free turns and captures. If Player 1 were to take all the free turns they possibly could, the win rate of Player 1 would skyrocket to 88.0%! Likely this is due to more seeds passing through Player 1's store since there are more free turns on Player 1. Interestingly, by giving the

simulations the ability to capture seeds whenever possible, the win rate reduces to 62.3%. So, this shows that the best strategy for Player 1 is the funnel opening with free turns.

Taking into account what the best strategy for Player 1 was, we went on to find the best strategy for Player 2. Against the funnel opening, Player 2's best moves are to get the free turn and defend against the possible capture by moving 21.

Move Number	**Player 1**	**Player 2**
1	13, 16	22, 21

Against Player 1's best strategy we determined just now, Player 1's win rate reduces to 56.6%. Thus, we went on to add free turns to Player 2's arsenal of strategies. This shoots down Player 1's win rate to 48.2%, 1% lower than Player 2's current win rate. Letting Player 2 capture whenever possible also reduces player 2's win rates, but only very marginally to 48.7%, which just so happens to be the same win rate as Player 1. This is the lowest we could bring Player 1's win rates by adjusting Player 2's strategies. It is safe to say that Player 2 does have a slight edge over Player 1, despite Player 2 technically having no possible openings since everything changes after Player 1 does their moves.

The table shown below is a summary of the strategies simulated and the win rate for Player 1 and Player 2. If both players are level, it would be a fair game.

No.	**Strategy Deployed (P1)**	**Strategy Deployed (P2)**	**P1 Win Rate**	**P2 Win Rate**
1	random	random	41.4%	52.4%
2	funnel opening	random	48.5%	45.5%
3	funnel opening , free turn	random	88.0%	9.6%
4	funnel opening, free turn, capture	random	62.3%	34.0%
5	funnel opening, free turn	funnel opening	56.6%	41.0%
6	funnel opening, free turn	funnel opening, free turn	48.2%	49.3%
7	funnel opening, free turn	funnel opening, free turn, capture	48.7%	48.7%

Research Question 2:

What is the longest possible game of Mancala?

There is no definitive way to answer this question. Thus, the first thing we did was to look at the many games we simulated and to find the longest one. The longest game out of our 140,000 simulations consisted of 37 rounds (rounds here referring to when both players finish sowing all their seeds and it is Player 1's turn again).

However, we felt that this was not the most possible number of rounds as this was likely unoptimised for our purpose, so we had to resort to coming up with a theoretical best method, which is to have both players keep activating the pocket furthest from their store unless that results in a free turn or a capture, in which the second furthest would be chosen and so on. Players 1 and 2 will perform the same moves if possible so that the number of seeds in each player's pocket is not as lopsided. Theoretically this will work since it reduces the number of seeds stored.

Later when experimenting with this game, it was found out that it would be better to activate the largest pockets when the pockets in the players' playing zone were running short in order to evade more accidental captures. With this following method combined with some stalling, the most number of rounds we could get after multiple games was 65 rounds.

Overall, the most number of rounds we could get in a single game is 65 rounds.

Research Question 3:

How can we make the game more complex and fun to play? After the modifications, how have the winning strategies from the original game changed?

The main flaw that we found in Mancala was that Player 2 had a slight advantage over Player 1. There was also the idea that Player 2 has no definite openings and is all based on Player 1, but we feel that this is more of a core idea of the game rather than a flaw.

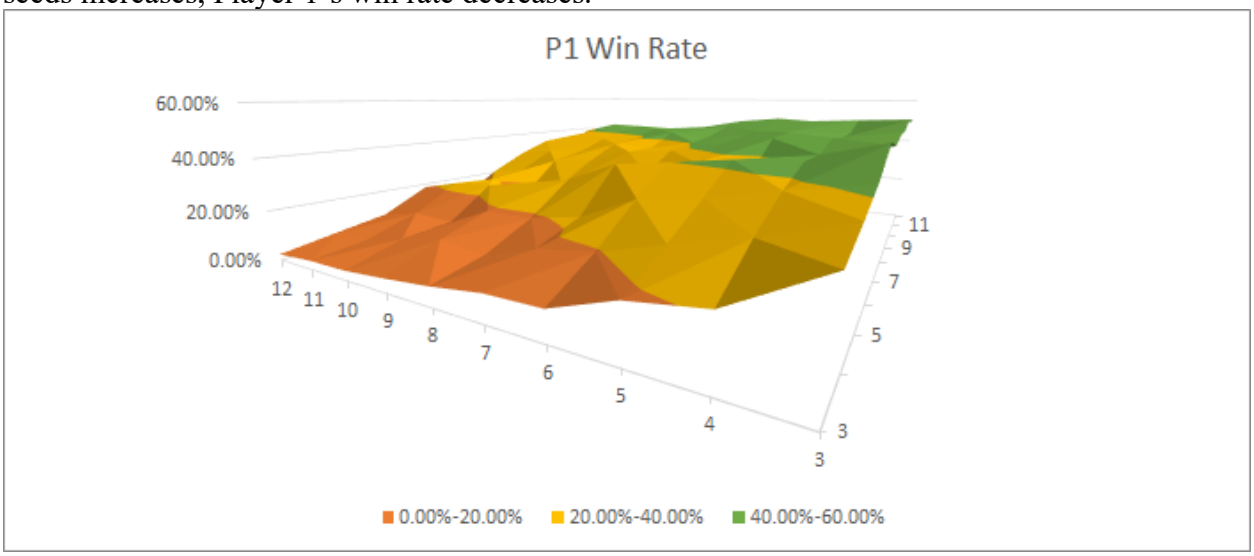
After some brainstorming, our group came up with the idea of having the ability to choose the direction of where the seeds go when the pocket is chosen - clockwise or counterclockwise. With this idea, the game could potentially turn out really interesting since there would be double the options possible. However, with this idea, there is the issue of games being able to be stalled for as long as possible in games where both players only have 1 seed left and can freely move it left or right. Also with some simulations, it was shown that the win rates were virtually the same where player 1 still has a slight advantage over player 2 with random moves, as well as the strategies being similar.

Another way we can modify the game is to add or remove more seeds or to change the number of pockets each player can access. Below here is a table which shows the win rate of player 1, as well as the win rate of player 2 after player 1's win rate so that the draw rate is shown if you

were to change the number of pockets and the number of players. Due to the number of games, the simulations won't be in the google drive and would only be done with 1000 games per suggestion.

		Pockets									
		3	4	5	6	7	8	9	10	11	12
Seeds	3	33.9% 55.9%	37.4% 55.1%	41.9% 50.2%	49.8% 43.3%	46.9% 44.4%	48.9% 44.2%	48.8% 44.1%	52.3% 43.3%	50.4% 43.5%	50.7% 44.7%
	4	21.1% 72.8%	30.5% 63.1%	37.9% 56.1%	43.1% 51.8%	42.8% 51.2%	46.9% 49.0%	45.5% 49.0%	45.9% 49.9%	50.7% 45.5%	49.0% 47.5%
	5	18.0% 76.7%	25.3% 69.0%	33.4% 62.4%	42.3% 52.1%	35.8% 5.88%	45.8% 51.4%	46.9% 49.5%	46.7% 50.4%	49.7% 46.1%	47.4% 46.9%
	6	10.5% 86.5%	19.5% 75.7%	30.9% 64.0%	39.9% 55.5%	37.1% 57.5%	42.0% 54.5%	42.8% 53.7%	45.2% 50.9%	47.8% 49.7%	49.7% 49.7%
	7	10.0% 87.3%	16.8% 78.9%	22.1% 74.6%	37.8% 57.7%	30.8% 65.7%	38.7% 58.2%	40.2% 56.7%	41.9% 54.1%	45.7% 50.8%	47.6% 50.0%
	8	7.3% 90.6%	11.0% 86.6%	18.8% 78.7%	30.0% 67.5%	26.3% 70.0%	36.8% 59.8%	36.7% 59.6%	43.1% 54.4%	43.3% 53.8%	43.6% 53.3%
	9	5.4% 93.4%	12.9% 85.0%	18.8% 78.0%	26.7% 69.7%	22.3% 74.1%	30.8% 66.9%	37.5% 60.1%	39.2% 57.7%	42.9% 54.8%	41.9% 56.1%
	10	3.8% 95.6%	9.6% 88.6%	11.4% 86.5%	23.6% 73.1%	15.9% 81.9%	29.5% 67.8%	32.5% 64.4%	37.6% 60.2%	41.2% 57.1%	42.7% 55.7%
	11	3.6% 95.4%	6.1% 93.0%	12.5% 84.6%	22.5% 74.8%	21.2% 76.8%	23.3% 73.9%	31.4% 66.3%	36.0% 62.2%	40.4% 57.3%	43.3% 55.1%
	12	2.3% 97.3%	5.8% 93.5%	9.2% 88.6%	18.5% 80.0%	9.6% 89.0%	20.4% 77.4%	28.3% 70.3%	34.3% 63.6%	31.5% 66.9%	38.3% 60.3%

Shown below is a 3D graph showing how the number of seeds and pockets affects Player 1's win rate. As the number of pockets increases, Player 1's win rate increases. As the number of seeds increases, Player 1's win rate decreases.



This modification could be used to give either player a "handicap", or even to make the game more fair. The number of seeds and pockets which are the most equal are 12 pockets and 6 seeds, though any combination of pockets and seeds that has a difference of 2% or less would also be considered a fair game. Compared to the rest of the choices, the typical game of mancala would be considered fairly fair as well. Note that pockets and seeds of only 3 to 12 are used to reduce the amount of time simulating and since these would probably be the most fun and realistic numbers used for games, with not too many seeds and pockets to make it confusing as well as not too little seeds and pockets to make it too boring.

Conclusion

In conclusion, here are our results for all three of our research questions.

What is the best strategy to win a game of Mancala? For Player 1, the best strategy we found is to start off with the Funnel Opening, which was mentioned earlier, and to take free turns whenever possible. For Player 2, the best strategy that we found to combat Player 1's best strategy is to respond defensively to the Funnel Opening as such:

Move Number	**Player 1**	**Player 2**
1	13, 16	22, 21

If both players play the best moves possible in each scenario, in theory, the game would be a fairly even game, with each player winning an equal number of games against one another.

What is the longest possible game of Mancala? The longest game of Mancala we were able to find was a total of 65 rounds, with the use of certain strategies such as delaying captures and storage, and prioritising large pockets to avoid accidental captures.

Finally, what modifications did we suggest to make to the game of Mancala to make it more enjoyable for each player? Firstly, we suggested the option of choosing which way the seeds would be sown in after activating a pocket. This would encourage more creative and complex gameplay, making the game less predictable and even more skill-based. Second, we suggested adjusting the initial number of seeds within the pockets, as well as the total number of pockets, which, after testing, we found changed the results of the game rather drastically.

Extension of the Project:

The shortest game of Mancala is 4 rounds long, with the move pattern being 13162223112413251126. This game is so short since Player 1 captures Player 2's seeds at round 2 after a successful funnel opening, leaving 3 pockets of seeds at the front for Player 2 to activate in order while Player 1 stalls. Such a game might happen if you are playing with someone who is completely new to Mancala, especially the early game capture which utilizes the funnel opening.

References

Seth Brown (2020). Tips to win Mancala. Retrieved from https://www.ultraboardgames.com/mancala/tips-how-to-win-mancala.php