The Math Behind Mancala

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1 Introduction and Rationale

Mancala is a game that is widely played in Africa and Southern Asia, and areas influenced by African or Asian cultures. It involves competition between two players in the distribution of pieces (beans or pebbles) into rows of pits and stores, under various rules that permit the accumulation of pieces by capture. Our group came across this ancient game, Mancala, and we found it very interesting as it was part of the ancient African culture then that is rarely researched. Due to its rich history, there are hundreds of variations of Mancala. For this project, we will be researching the basic two-tier Mancala or Kalah.

2 Mancala Rules

Basic Rules:

Fig. 1 Mancala board

(Image is taken from https://www.funlearning.co.uk/wp-content/uploads/2018/04/LAG0216_Lagoon_Mancala_C_1.jpg)

As depicted in *Fig. 1* (above), there are 12 pits, 6 on each side. There are 2 stores, one yours, one for your opponent. The objective of the game is to have the most seeds in your store.

Starting the Game:

- Place 4 seeds in each pit.
- On a turn, a player picks up all the seeds in one pit and "sows" them to the right, placing one seed in each of the pits along the way

Special Rules:

- When the last seed in your hand lands in your store, take another turn.
- When the last seed in your hand lands in one of your empty pits, you get to steal all of the seeds in your opponent's pit on the opposite side.

Ending the Game:

- The game is over when one player's pits are empty. The other player takes the seeds remaining in her pits and puts those seeds in her store. Count the number of seeds. Whoever has the most seeds wins.

3 Objectives

1a. To investigate the probability of the first player winning

1b. To investigate the probability of the second player winning

2. To explore algorithms used to play Mancala

3. To find out the length of the game (R.M.) on average

4 Research Questions

1. Which player has the advantage in a game? (P.M., R.M.)

2. What are the algorithms used to play Mancala?

3. How many possible games of mancala can be played?

5 Field of Math

- Probability and Statistics

6 Terminology

Term	***Explanation***
Two-tier Mancala	It is the variation of Mancala that is played with two rows of pits, 6 pits in each row, with 1 store at the side.
Randomized Moves (R.M.)	They are the moves that both players play at random where they have no intention of winning the game and are not strategizing at all. When recording moves for an R.M. game, when the random die roll shows a pit with 0 seeds, we ignore that roll.
Playing Moves (P.M.)	They are the moves when both players play to win the game for themselves.
End of 1 Move	It is when 1 player cannot move any more pieces, and the chance to play is passed on to the opponent.

7 Methodology

I. Experiment with some gameplay of Mancala

(Record down moves, numbers of moves, etc)

II. Read up on books relating to probability and statistics

III. Read up on articles (online) on Mancala (specifically Kalah),

IV. Calculate the answers to the research questions.

8 Literature Reviews

In 2000, Donkers etc. researched Mancala, focusing on the aspect of Human versus Machine Problem Solving. In 2013, Divilly etc. explored and analyzed the evolution of strategies for Mancala variants. Their research aimed to evaluate the performance of a collection of heuristics across a selection of Mancala games. However, there are barely any existing works of literature which touch on the winning strategies of Mancala, a gap which our proposed research can contribute to filling in.

9 Results

1st Research Question:

∵ After playing 100 R.M. games, Player 1 wins 46 times, Player 2 wins 47 times and there were 7 draws.

- Chances of Player 1 winning is 46%,
- Chances of Player 2 winning is 47%.
- Chances of drawing are 7%.

∴ Chances of either player winning are roughly equal if moves are played randomly.

2nd Research Question:

- After many searches online, no algorithms used for playing Mancala
- Moves played are mostly a reaction to the moves played by the opponent
- However, there are a few general strategies that we found, which can be seen in the next few slides

STORE 2	2F	2E	2D	2C	2B	2A	STORE 1
	1A	1B	1C	1D	1E	1F	

Table 2: Set up for labelling each pit and store.

Strategy 1:

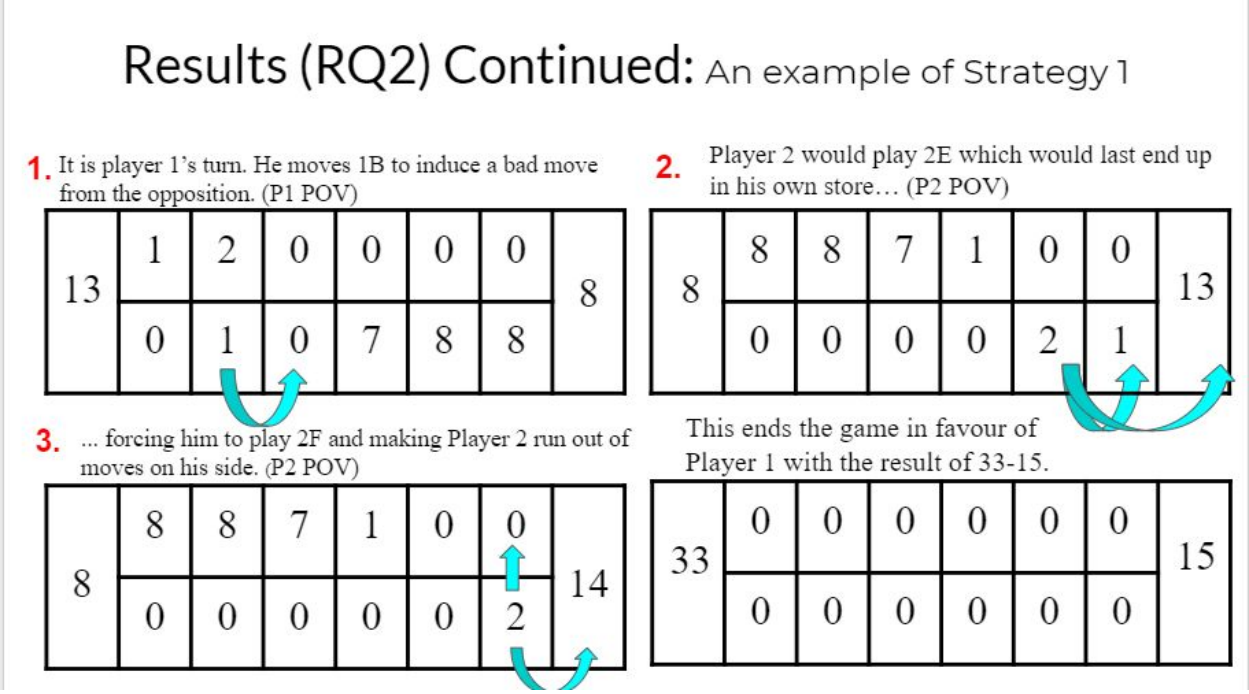

Fig. 3: Example of Strategy 1

As depicted in Fig 3 (above), the first strategy is to:

- Stack seeds in a few pits so that the opponent has no choice but to play a bad move as it is their only option
- **Example:** It is Player 1's turn. He moves 1B to induce a bad move from the opposition. Player 2 would play 2E which would end up in his store, forcing him to play 2F and making Player 2 run out of moves on his side. This ends the game in favour of Player 1 with the result of 33-15.

Strategy 2:

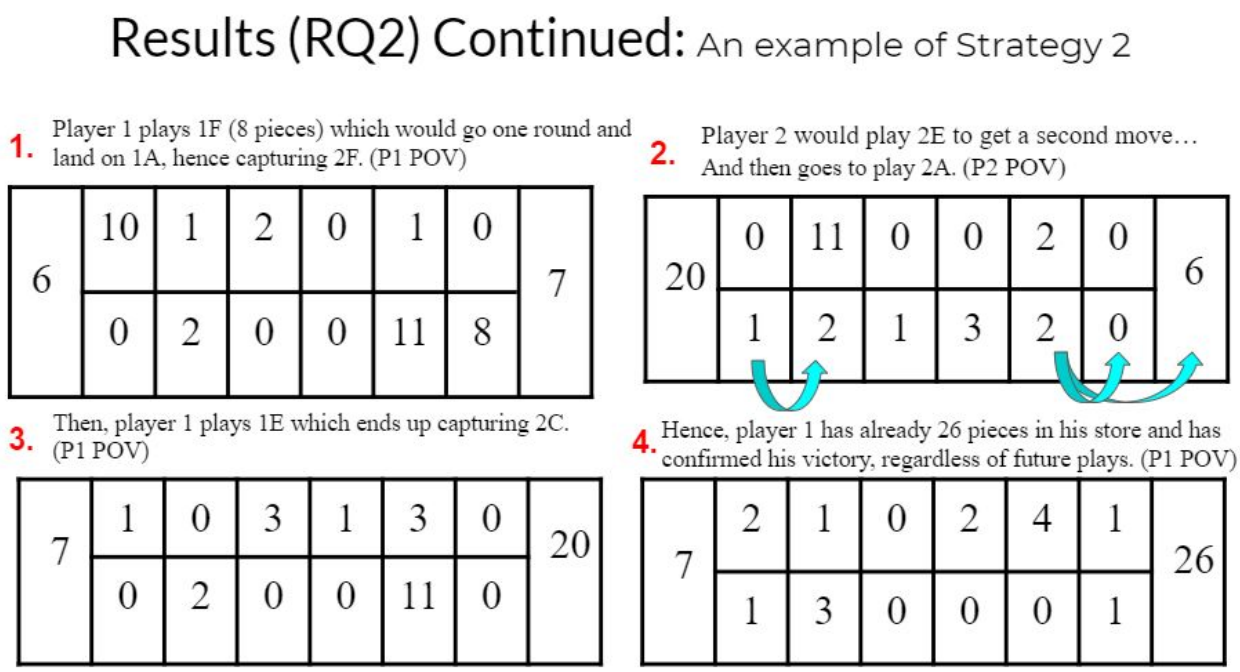

Fig. 4: Example of Strategy 2

As depicted in Fig 4 (above), Strategy 2 is to:

- Stack a high number of seeds in the pits closest to your store
- Letting them travel one round around the whole board
- Landing on an empty pit opposite to the opponent's, "stealing" all of the seeds.
- **Example:** As seen in Fig 4. Player 1 first plays 1F which contains 8 seeds, which would go one round and land on 1A, hence capturing 2F.
- The second player would play 2E as the last piece lands in the store, and he would get a second move.
- Later, he would play 2A (random move).

- Next, the first player who has stacked 11 pieces in 1E, plays 1E. This allows him to capture 3 pieces from the opponent's 1C. Now, he already has 26 pieces in his store and is guaranteed victory.

Strategy 3:

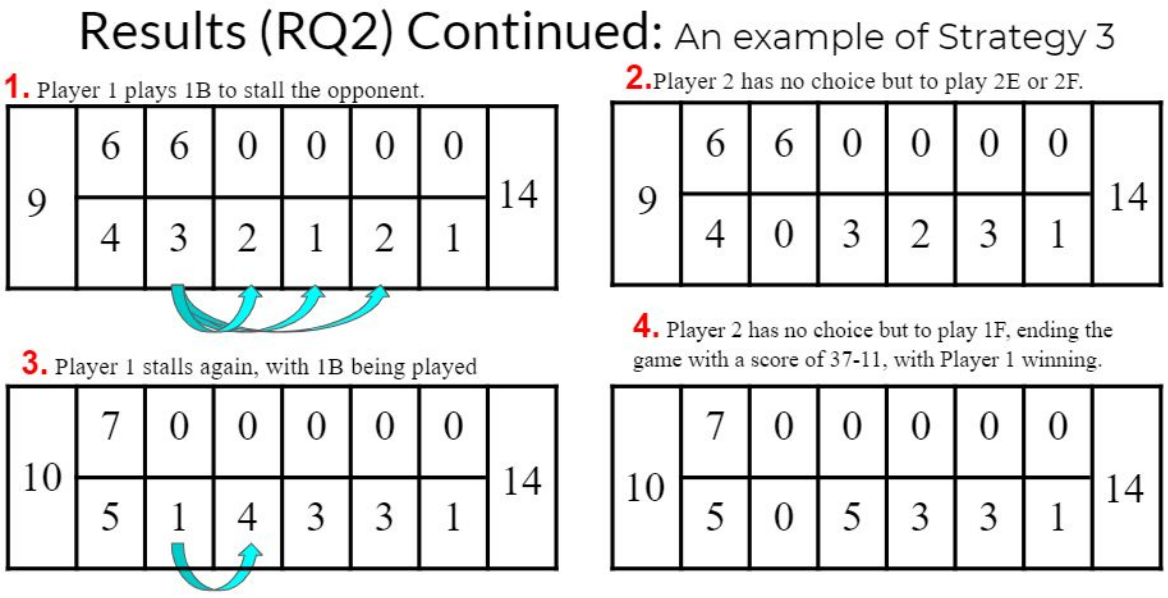

Fig. 5: Example of Strategy 3

As depicted in Fig. 5 (above), our third strategy is to:

- Spread the seeds out and play them one by one. One player tries to have as little seeds in a pit as possible, yet having all 6 pits occupied with seeds most of the time.
- With this method, you would need to stall the opponent. To do so, you have to delay by playing the pieces which can spread out.
- The opponent has to play his stacked up pieces, and that frequently results in a loss as their stacked up pieces do not have enough seeds to go one round around the board, which in the end, causes them to run out of pieces on their side.

- **Example:** As a stalling move, Player 1 plays 1B. Since player 2 has only 2 stacks left, he is forced to play either 2E or 2F, in which he would play 2E.
- To continue stalling, player 1 plays 1B again, as there is only 1 seed in that pit. Because of this, player 2 has no more alternatives and is forced to play 2F, which only has 7 seeds, hence being unable to have seeds go back to his side of the board.
- Player 1 would take all the seeds in his side of the board into his store and win the game.

3rd Research Question:

- After playing 100 R.M. games, we found out that the average length of each game is about 37.61 or 38 (nearest whole number)
- After playing 25 P.M. Games, and we found out that the average length of P.M. games with our research is about 18.96 or 19 (nearest whole number)
- By counting the number of possible pits to play per move for the 10 R.M. games with the highest number of moves (for greater reliability)
- We estimate that the average number of possible pits that can be played per move is around 4.58 (3s.f.).

∴ Estimated that the number of possible Kalah games is around 4.58^38 using the formula: Number of possible games that can be played = The average number of possible pits that can be played ^ The average length of each game.

Justification:

Number of Games:	Mean:	Median:
10	38.8	38.5
20	36.6	38.0
50	36.4	37.0
100	37.6	38.0

Fig.6: A table showing our results of the average lengths of R.M. Games

As seen in Fig.6 (above), even though we only played 100 R.M games, the mean and median moves are very consistent.

10 Conclusion

- Chances of either player winning is roughly equal when played randomly.
- **Algorithm 1:** Stack seeds in a few pits so that the opponent has no choice but to play a bad move as it is their only option
- **Algorithm 2:** Stack a high number of seeds in the pits closest to your store, letting them travel one round around the whole board, landing on an empty pit opposite to the opponent's, "stealing" all of the seeds.
- **Algorithm 3:** Spread the seeds out and play them one by one. One player tries to have as little seeds in a pit as possible, yet having all 6 pits occupied with seeds most of the time. With this method, you would need to stall the opponent. One can stall by playing the pieces which can spread out. The opponent has to play his stacked up pieces, and that

frequently results in a loss as their stacked up pieces do not have enough seeds to go one round around the board, which in the end, causes them to run out of pieces on their side.

- Average length of games (R.M.) is 37.61 or 38 when rounded off to the nearest whole number.
- Average length of games (P.M.) is 18.96 or 19 when rounded off to the nearest whole number.
- Average number of possible moves that can be played per move (R.M.) is 4.58
- Estimated total number of possible games is 38 ^ 4.58 or 1.296845e+25.

11 Limitations

Our results were drawn from only 100 games, we could have played more than 100 R.M games but we had little time to do so due to the COVID situation and little face to face interaction due to the CB period.

12 Project Extension

We wish to program a Mancala engine that will play more games and can play the best moves. This will allow us to obtain more reliable results.

13 Timeline

Event/Deadline	*Date*
Start of Project	EST 14 Feb
Start of Written Report	17 Feb
Start of Project Proposal Slides	3 Mar
Forming of Research Questions, Methodology, Objectives, etc.	T1W7 - T1W10
Reading of Literature Reviews/Ideas	T1Hols
Begin forming answers to research questions, conducting experimental research	T2Hols
Refinements on the project, Preparation of Powerpoint for Finals	T3
Final Evaluation	12 August
Final Written Report	20 August

14 References

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