Rubik’s Cube Study

Hwa Chong Institution (High School)

Project Work 2020 - Category 8 (Mathematics)

Written Report

Group 8-21

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1.0 Introduction

The Rubik's Cube has been a very well-known toy for several years, challenging for most, but a piece of cake for the intelligent few. As of January 2009, 350 million cubes had been sold worldwide, thus widely regarded as the world's best selling toy. It is a 3D combination puzzle invented in 1974, by Ernö Rubik.

1.1 Rationale

The Rubik's Cube is not only a three-dimensional puzzle to toy with for fun, but also a source of mathematical concepts and calculations. Through this project, we intend to learn more about the mechanics of the Rubik's Cube, and get more in-depth knowledge about how it works and the mathematics behind it. We also aim to discover more about the different types of cubes, including studying their mechanisms and algorithms.

1.2 Research Questions

1) What are the factors which affect speedcubing?
2) What is God's number?
3) How are the algorithms formed?

2.0 Mechanics

A standard Rubik's Cube measures 5.7 cm on each side, which consists of two main parts, the core and the cubelets, which include the centerpieces, edge pieces and corner pieces. The edge pieces and corner pieces have a concealed inward extension which interlock with each other so as to move to different locations. The centerpieces are affixed to the core mechanism, as they have axles pivoting on a fastener held by a

“3D cross” consisting of 3 intersecting axes, in order to provide structure for the other cubelets to fit into and move around the centerpieces. Thus, enabling each face to turn independently by rotating the centerpieces.

2.0.1 Orientation of Colours

There is a standardised arrangement of colours so as to prevent any confusion or misunderstandings. Usually, the white is opposite yellow, red is opposite orange, and blue is opposite green. But there are also other orientations of colours which are rarely used when manufacturing the Rubik’s Cube.

2.1 Notations

The following moves are the moves and notations of the Rubik’s Cube, the basics that everyone needs to know when solving the Rubik’s Cube.

- R - Right (The face directly to the right of the front face)
- L - Left (The face directly to the left of the front face)
- U - Up (The face directly on top or above the front face)
- D - Down (The face directly below the front face)
- F - Front (The face that is facing the solver)
- B - Back (The face opposite of the front face)

- Non-capital letters refer to double-layered moves of that certain face (E.g - r)
- A ‘2’ following the letter means that the face has to be moved twice, or 180 degrees (E.g - R2)
- An apostrophe after the letter means that the face should be turned anti-clockwise instead of clockwise (E.g - R’)

Middle-layered turns

- M (Middle) - The middle layer between R and L
- E (Equator) - The middle layer between U and D
- S (Side) - The middle layer between F and B

Cube rotations

- x - Rotate the cube on the x axis
- y - Rotate the cube on the y axis
- z - Rotate the cube on the z axis

2.2 Intended Methodology

Specific methods are required to solve those research questions. We dismantled a cube, used Mathematical algorithms to find the algorithms of solving a cube and trying them out to solve the cube.

3.0 Literature Review

“The fastest people need about 50 moves to solve a Rubik’s Cube. Our AI takes about 20 moves, most of the time solving it in the minimum number of steps,” said the study’s senior author, Pierre Baldi, UCI distinguished professor of computer science. This clearly shows how an AI’s form of reasoning is completely different from a human’s, as the study clearly shows that the AI’s strategy is different, compared to a human. Cubers should learn to adopt that method, especially in the WCA (World Cube Association) event FMC (Fewest Moves Challenge), where competitors are supposed to solve the cube in the fewest moves possible.

Jessica Fridrich is a professor at Binghamton University, who specializes in data hiding applications in digital imagery, including steganography and steganalysis, forensic analysis of digital images (sensor fingerprints) and advanced image processing. She is also known for documenting and popularizing the CFOP method or

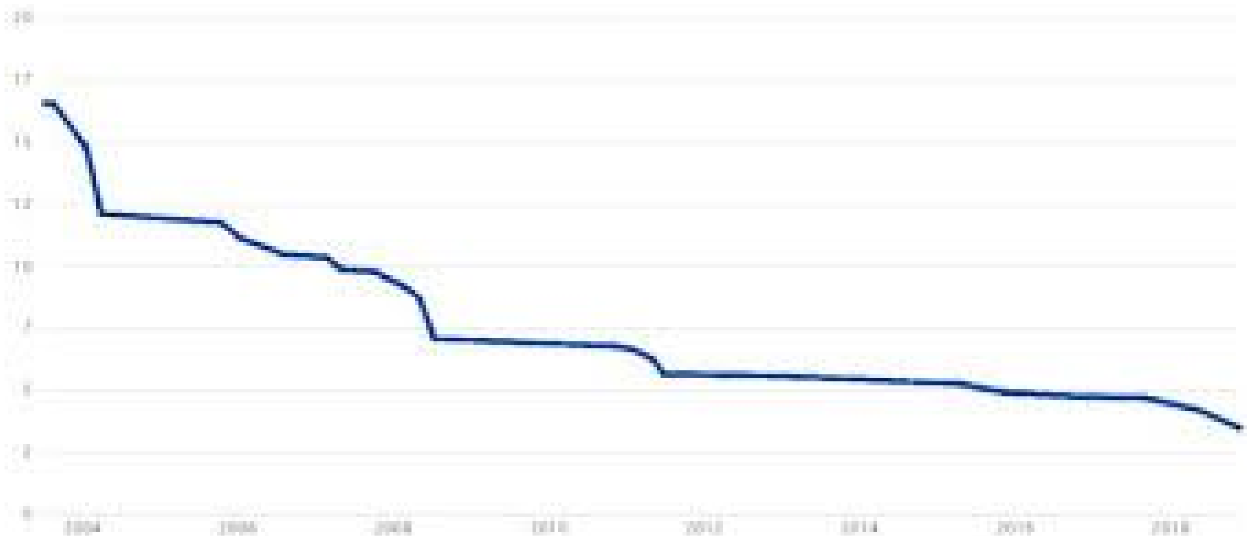
Fridrich method, one of the most commonly used methods for speedcubing (Cross, First 2 Layers, Orient Last Layer, Permute Last Layer). The method describes solving the cube in a layer-by-layer fashion. First a "cross" is made on the first layer, consisting of the centerpiece and four edges. The first layer corners and edges of the second layer are put into their correct positions simultaneously (four pairs). The last layer is solved by first orienting and then permuting the last layer of the cube using a few sets of algorithms.

3.0.1 History of Rubik's Cube

This highly attractive toy was founded by Erno Rubik, who was born in Budapest, Hungary on the 13th of July, 1944. He combined the divergent talents of his parents, and became sculptor and architect. Fascinated by the concept of space, Rubik also spent his free time then designing puzzles that would open his students' minds to new ways of thinking about three-dimensional geometry. In 1974, the cube was invented. Erno Rubik finally finished his prototype of the "Magic Cube", its previous name before it was named after its creator. He could not figure out how to solve his own creation and even spent one month trying to figure it out. In 1975, Erno Rubik obtained the patent for the "Magic Cube". It was named Rubik's cube 5 years after the patent. In 1977, the first batch of Rubik's cubes were sent to local toy shops for sale. In 1979, the world wide distribution deal was signed for the cube to be exported. Then, people all over the world finally got to see the Rubik's cube in 1980 and it became the top selling toy with Patrick Bossert publishing a book "You Can Do The Cube" which sold a whopping 1.5 million copies. In 1982, the First Rubik's Cube World Championship was held in the Capital of Hungary and the winner was Minh Thai, who solved the cube in 22.95 seconds. He then later published a book entitled "The Winning Solution", which provided tips on how to solve the Rubik's cube. In 1997, the Fridrich Method was published by Jessica Fridrich, it is regarded as one of the best speedcubing method, and the world record single and average times have been held by Fridrich solvers for many years, despite Fridrich herself estimating that the method would not be efficient for consistent times under 13 seconds.

3.1 Background

Speedcubing is about solving the Rubik's Cube as fast as possible. The speedcubing trend shows us the improvements made in terms of speedcubing and most importantly about human limitations.



The graph above shows the speedcubing trend for 3x3 Rubik's Cube Singles in WCA competitions. The first 3x3 world record was set at 22.95 s in 1982, at the first Rubik's Cube World Championship. Due to findings of new methods, algorithms and finger tricks, the record kept on improving especially from 2004 to 2008, where there were many drastic improvements, unlike recent years. It was till 2019, where Yusheng Du broke the record with a shocking timing of 3.475 s. The recent decrease in improvements was due to human limitations. In time to come, beating the world record would start to heavily depend on luck, as in every WCA event, every scramble is randomised and hence luck-dependent. Eventually, a record 3x3 single timing under 3 seconds will soon be possible with a lucky solve, where 1 or even 2 steps can be skipped.

4.0 Findings

Our findings took numerous attempts of trial and error. Through our attempts, we have found out more factors which affect the speed at which the Rubik’s Cube is solved, and steps to solve the Rubik’s Cube more efficiently and quickly.

4.1 Factors Affecting Speedcubing

There are a few factors affecting speedcubing which are important to consider. Firstly, we must account for how the cube was constructed.

- Lubrication
- Gears
- Corner Cutting
- Magnets

The lubrication and gears of the cube both affect how quickly and smoothly each face can turn, as they affect how much friction is acting between the axles. The gears should be smooth to an extent that the faces can be turned effectively, but not too smooth as it would be very easy to overturn the faces, which might cause locks during one’s solve, and thus a possibility where the cubelets burst out of their positions, which is known as ‘popping’. Locks happen when the pieces get caught on each other and cannot easily realign themselves.

Corner cutting is essential for speedcubing as it comes into play when one side of the cube is not turned fully. You can then turn the cube to the perpendicular side, allowing each move to be executed quickly. When solving the cube, one is bound to overturn or instead might not turn enough.

Magnets inside the cube are helpful as it ensures that each face turns to exactly in multiples of 90°, so that locks or jams in the cube would not happen when one is solving it.

Another factor is the method used to solve the Rubik's Cube. There are several methods to solve the Rubik's Cube, but these are some common methods which most cubers use.

- Layer-by-Layer
- CFOP
- Corners First
- ZZ
- Roux

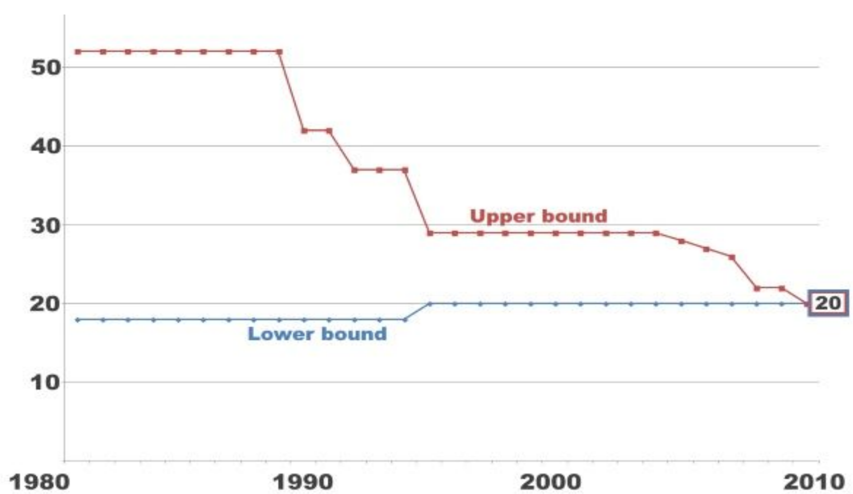
We will look further into these methods in the third research question.

4.2 Discovery of God's Number

God's number is 20, which is the maximum number of moves needed to solve any of the 43 252 003 274 489 856 000 permutations of a Rubik's Cube. This big number is derived from the equation $\frac{1}{2} \times (8! \times 3^7) \times (12! \times 2^{11})$. There are 6 fixed centers, but 12 edges and 8 corners which are moveable. There are 8 ways to arrange the 8 corners, so 8! = 40 320, and there are 3 possible configurations of a corner, and only 7 corners can be orientated independently, so 3^7. Next, there are 12 ways to arrange the 12 edges, so 12! = 479 001 600, and there are 2 possible configurations of an edge, and only 11 edges can be orientated independently, so 2^{11}. Thus, the formula

½ x (8! x 3^7) x (12! x 2^{11}). Here's another example where we implement the same idea on a Square-1. There are a total of 24 wedge pieces on the puzzle. Any permutation of the wedge piece is possible, including even and odd permutations, so 24! = 620 448 401 733 239 439 360 000.

As mentioned above, all permutations of the 3x3 Rubik's Cube can be solved in ≤20 moves, as it was conjectured that Super Flip would be the most difficult permutation to ever solve, where every single edge is flipped once, and it takes exactly 20 moves to solve.



From the picture directly above, there is a lower bound and an upper bound. The lower bound refers to the least number of moves needed to solve the worst case scenario while the upper bound refers to the most number of moves needed to solve the worst case scenario. Over the years, many new difficult and complex permutations have been discovered and the two bounds end up meeting at the number 20, which turns out to be God’s number.

4.3 Formation of Algorithms

There are many types of methods with different solutions to solve the cube, but there is one method which is commonly used among many speedcubers, The CFOP Method, also called the Fridrich Method.

C - Solve the white cross

F - Solve the first 2 layers

O - Orientate the last layer

P - Permute the last layer

Commutators and conjugations are very commonly used in these algorithms. The commutator of X and Y is X Y X’ Y’, and here are some examples of commutators

- Flipping two edges

X Y X’ Y’ = (L E’ L2 E2 L) U (L’ E2 L2 E L’) U’

- Rotating two corners

X Y X’ Y’ = (F’ D F L D L’) U (L D’ L’ F’ D’ F) U’

- Cycling three corners

X Y X’ Y’ - (L D L’) U (L D’ L’) U’

Conjugations are also commonly used where the conjugate of X is Z X Z’, here are some examples of conjugations.

- Cycling three top edges

Z X Z’ = (F2 U) M U2 M’ U2 (U’ F2) = F2 U M U2 M’ U F2

- Cycling three top corners while preserving orientation

Z (X Y X’ Y’) Z’ = F’ L F’ R2 F L’ F’ R2 F2

With the help of commutators and conjugations, many algorithms can be formed to solve the many different cases and scenarios, and here are some examples.

- Inserting edges (F2L)

X Y X’ Y’ A B A’ B’ = U’ L’ U L U F U’ F’ → Where 2 commutators are combined

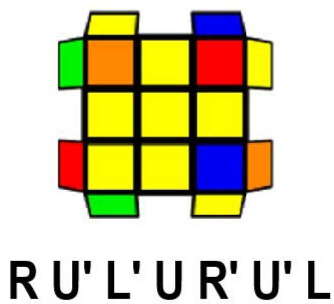
- ‘Fish’ algorithm (OLL)

Z X Z’ = (R U) (R’ U R U2) (R’ U’)

- U-perm (PLL)

Z X Z’ = (F2 U) M U2 M’ U2 (U’ F2) = F2 U M U2 M’ U F2

Also, there are short and simple algorithms which can be used within other longer ones. For example, the sexy move, sledgehammer, and air jeff.



Algorithms in COLL are algorithms from OLL and PLL of CFOP combined, which increases in efficiency, but might be harder to execute. As you can see in the picture above, the COLL algorithm is relatively short, but the CFOP algorithm comprises a “Fish” algorithm and a J-perm.

5.0 Conclusions

Through this project, we discovered that cubers solve the Rubik’s Cube using algorithms which are formed by and associated with mathematical concepts and equations. We also found out that even speedcubing has many factors which are required to be accounted for. The fast and rapid increase in the Rubik’s Cube records require in-depth studies of various new methods and algorithms over a long period of time. We also knew that Mathematics is all around us, and is especially used in the discovery of new methods and possibilities. We found this study helpful in boosting our understanding of mathematics.

6.0 Possibility of Project Extension

If we were to explore more into the world of Rubik's Cubes, we could study more about algorithms which are used in blindfolded solves, fewest moves challenge and one-handed solves, and also more complex cube variants including Square-2, Axis cube and Ghost cube. During blindfolded solves, one is supposed to memorise the position of every cubelet on the Rubik's Cube, and by using only 1 algorithm, repeat it many times to bring cubelets to their original locations. For the fewest moves challenge, cubers are given a period of time to figure out the least possible moves to solve that permutation, which requires a lot of trial and error, and it is similar to how AI solves the cube. Lastly, one-handed solves are all about efficiency, as that one hand is gripping to that certain side of the cube so moves are usually executed on the other side of the cube. For example, if one is using their left hand to solve, R and U moves are usually executed as it is easier, and thus algorithms have to be changed.

7.0 References

Speedcubing graph
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