<u>FOX AND SHEEP by GROUP 8-20</u>

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(1) Introduction and Rationale

1.1 Why do we want to do research on this game?

We are trying to find a strategy for this fox and sheep game. We hope we can find a way to win this game. In our opinion, this is a particularly interesting game to research on since this game has so many ways to win or lose and it would definitely be exciting if we could find one or two permutations that allow us to win. This game is generally unpopular, so we hope that through this project, we can raise awareness about this game and make it more popular.

(2) Literature Review

2.1 History of the game

The game, previously known as *Halatafl,* is known from as early as the 14th century. It most probably originated in Scandinavia. This game was a favorite pastime of Queen Victoria. The earliest uncontroversial record of fox & geese (the original Fox and Sheep game) is from the accounts of English King Edward IV (1461-1470 & 1471-1483). Since then, there have been many variants of the game.

2.2 Rules of the game

There will be a total of 33 points in the shape of '+'. There will be 20 sheep at the bottom of the board and 2 foxes at the top right and top left of the board game. Any sheep or fox cannot be in the same box. A sheep can only move up, right, down or left one box every turn while the fox can move in all directions (but have to follow the lines). The objective of the fox is to try to eat 12 sheep by jumping over a sheep.

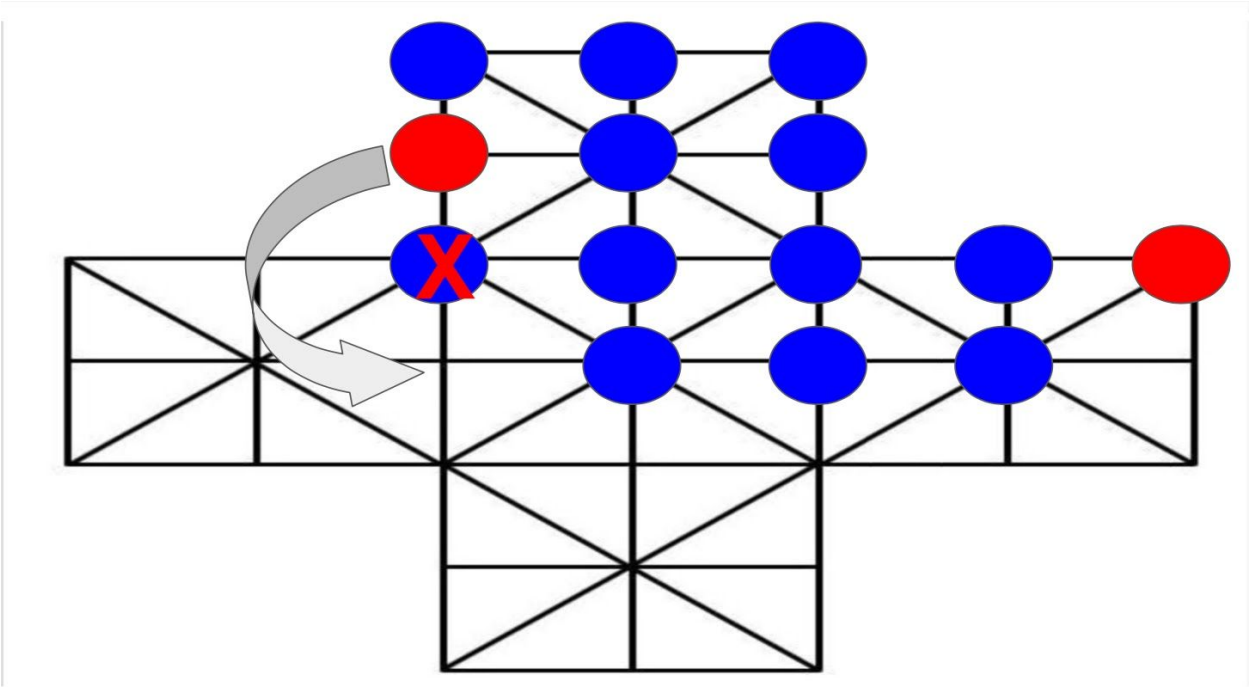
**A fox cannot jump over two or more sheep.*

The objective of the sheep is to try to have 9 sheep to cross over to the other side of the board while avoiding getting eaten by the foxes on the way. If 9 sheep manage to cross over to the winning zone, then the sheep win. If the fox eats at least 12 sheep before the sheep wins, then the foxes win.

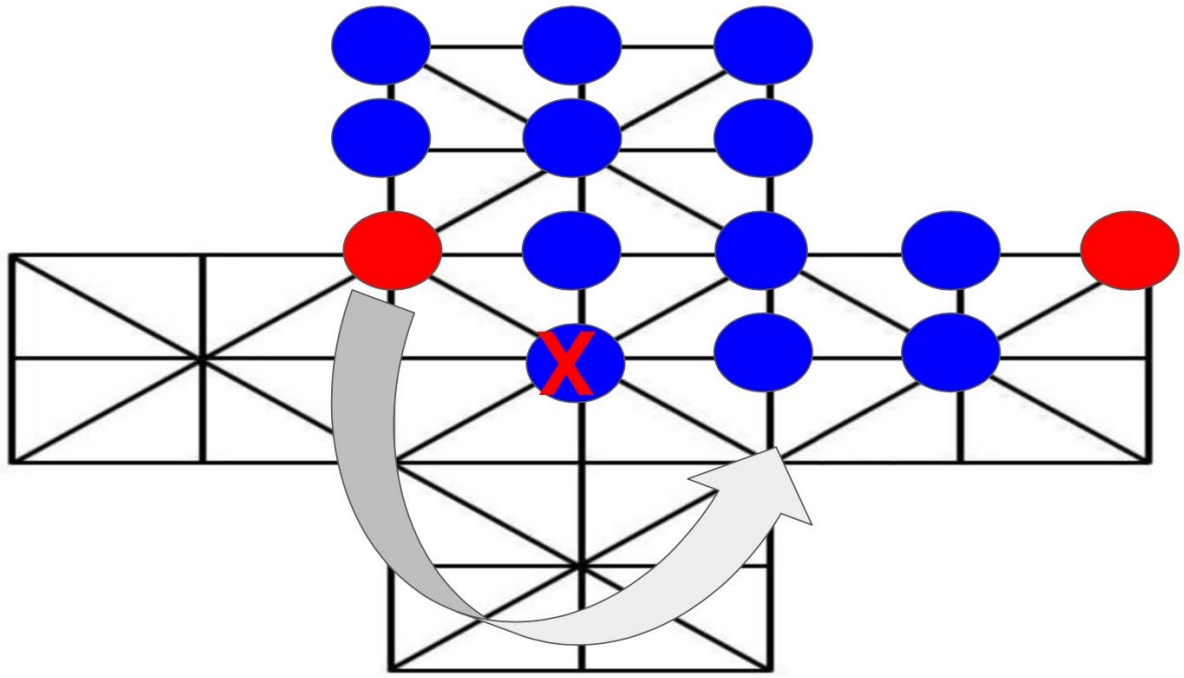
2.2.1 Two examples of a fox eating a sheep.

**For all diagrams, blue represents a sheep while red represents a fox.*

1st example:

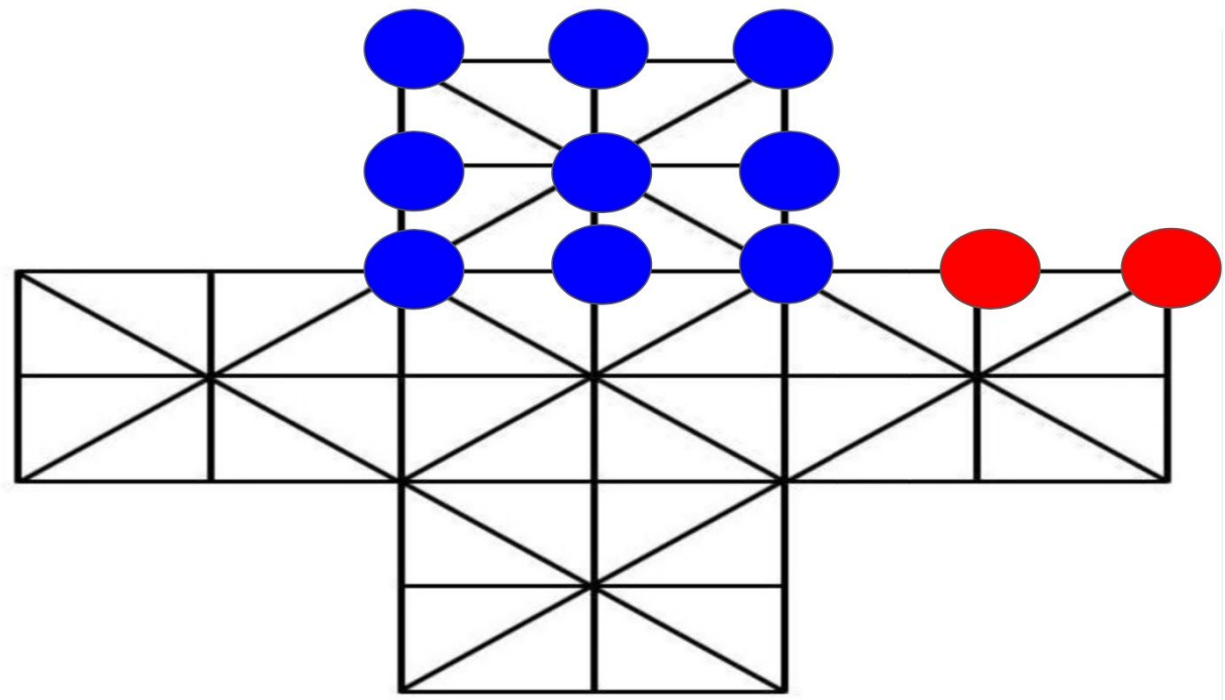


2nd example:



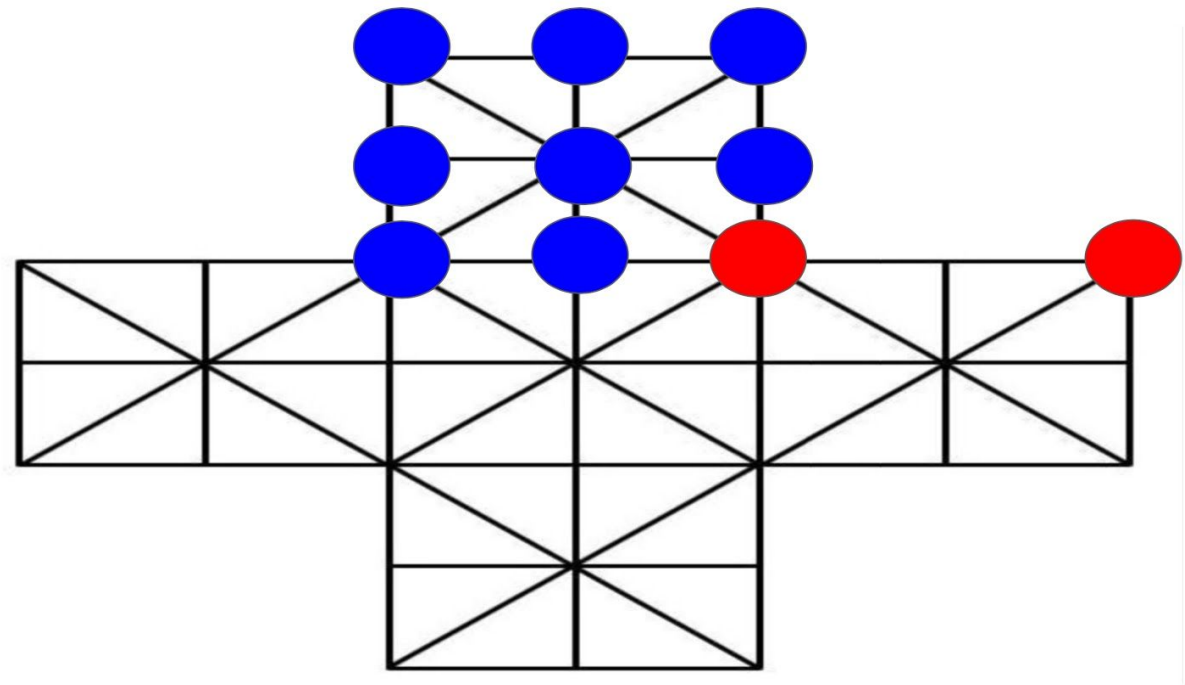
In both cases, the sheep that is eaten (represented with a red ‘X’) is removed from gameplay.

2.2.2 Example of sheep winning



Reason: There are 9 sheep in the winning zone.

2.2.3 Example of foxes winning



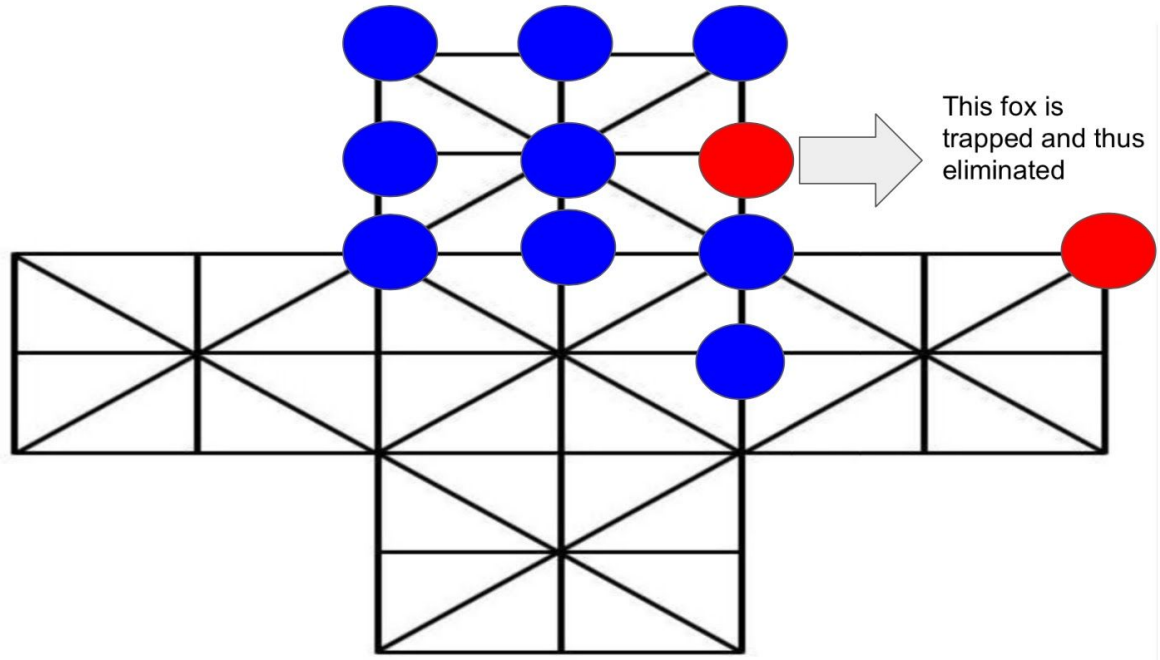
Reason: There are less than 9 sheep left

2.2.4 Fox getting trapped

If the sheep surrounds a fox such that a fox has nowhere to go, that fox will be eliminated from gameplay, as it has been overpowered. When both foxes are eliminated from gameplay, the sheep win.

Note: Foxes can only be eliminated right after a sheep player's turn.

2.2.4.1 Example of a fox getting trapped.



2.2.5 Deadlock

Inevitably, deadlocks may occur sometimes, whereby 1 fox stays in the winning zone and another fox stays outside and eats the sheep, causing a repetition of moves in both

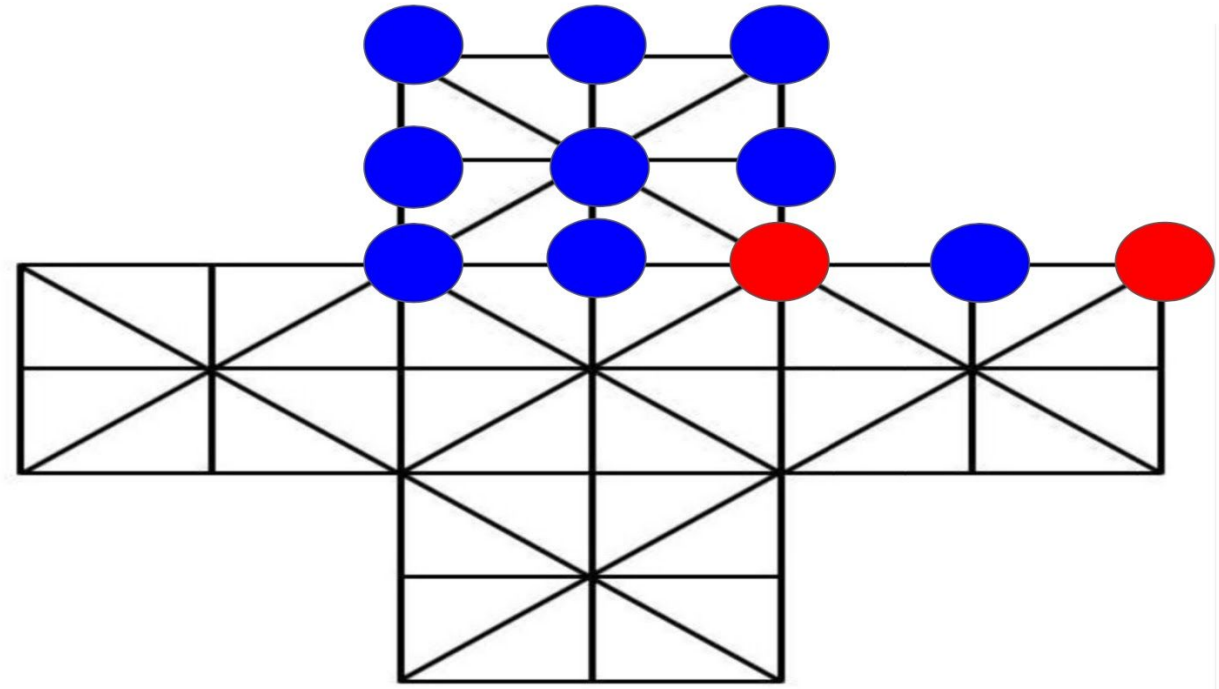
the foxes and sheep. In such a case, then if there are 14 or more sheep, the sheep win. Otherwise, the foxes win.

Hence, rarely do the sheep ever win in the event of a deadlock.

However, deadlocks cannot come into gameplay if there is only 1 fox left, except in Example 2, as shown below.

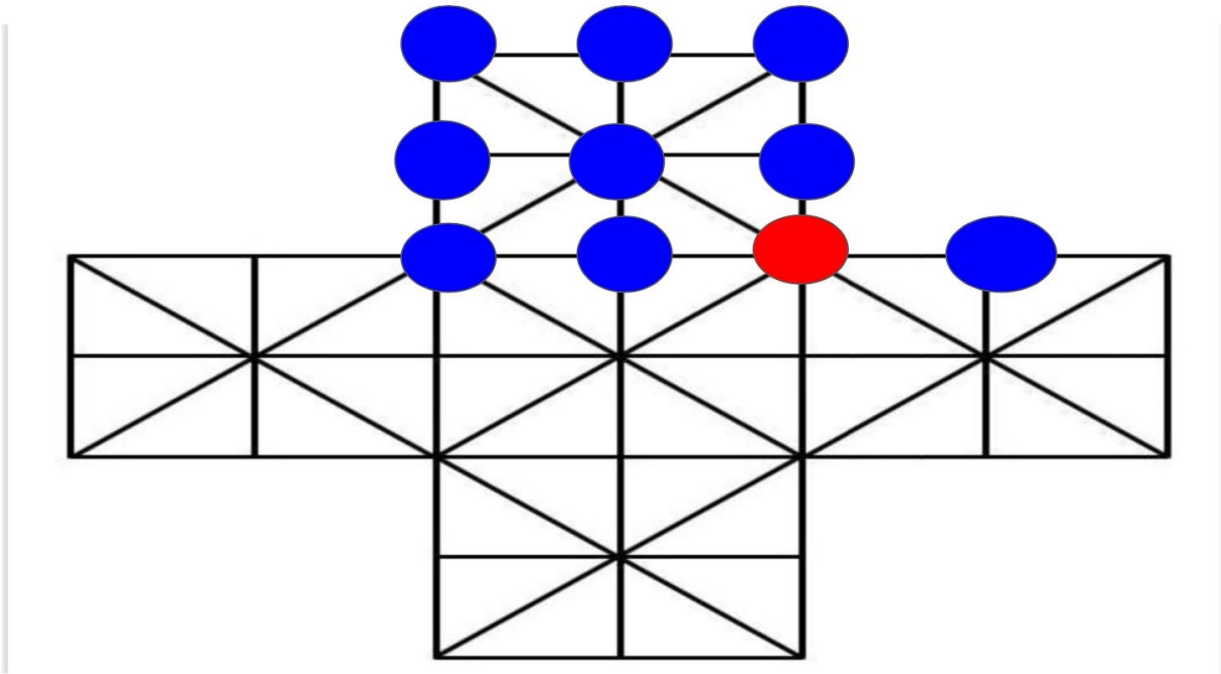
Two Examples of a deadlock

Example 1:



In such a case, the fox wins.

Example 2:



Note: This is a special case, because as mentioned earlier, deadlocks are not considered when there is only 1 fox left. In such a case, the foxes win.

2.2.6 What strategies are there currently?

There are , unfortunately, no strategies to this game so far. This is unsurprising, considering its unpopularity. That is why our objective that was mentioned later is as such, we aim to find strategies for this game and in the process make it more popular.

(3) Intended Methodology

Our intended methodology for this project is to do it by tabulation, using an Excel document. This is because tabulation is one of the easiest and most efficient ways of finding out whether a strategy works or not. Other methods may involve complicated things like coding, but tabulation does not.

(4) Objectives and Research Questions

4.1 Objective

Our objective of this project is to find out what happens when the map changes, or even the number of foxes or sheep and also raise awareness for this game at the same time. Below, are the answers, in full, to our research questions.

4.2 Research Questions

4.2.1 Question 1. Exploration of different strategies for this game.

Unfortunately, we were unable to find any strategies on the Internet. However, we did introduce some strategies to this game.

<u>The strategies for the sheep:</u>

Sheep strategy 1(Sheep 1):

Aim to trap foxes by sacrificing some sheep. Basically, some of the sheep will allow themselves to get eaten, so that other sheep can trap the fox as the fox would be in a bad position.

Sheep strategy 2(Sheep 2):

Aim to have the sheep stay as close as possible, as foxes cannot jump over multiple sheep.

Sheep strategy 3(Sheep 3):

Aim to get to the winning space as fast as possible (similar to dashing in regardless of consequences).

Sheep strategy 4(Sheep 4):

Sacrifice some sheep. Then, with the fox in a bad position, slowly get the sheep into the winning space. The difference between this strategy and Sheep 1 is that this strategy aims to win by getting 9 sheep in the winning zone, but Sheep 1 aims to trap the fox instead.

The strategies for the fox:

Fox strategy 1 (Fox1):

Aim to keep at least 1 fox in the winning space in order to present threats to the sheep when they enter the winning space.

Fox strategy 2 (Fox2):

Aim to corner multiple sheep.

Fox strategy 3 (Fox 3):

Allow some sheep to come into the winning space. Then, Aim to leave the sheep in a trap such that no matter what, at least one sheep will get eaten.

What have we done?

To calculate the effectiveness of each strategy, we have compared them with each other by tabulating. Here is the table to our results, with a red (or a -1) representing a fox winning, while a green (or a 1) representing a sheep winning.

(Sheep, fox)		1,1	1,2	1,3		2,1	2,2	2,3		3,1	3,2	3,3		4,1	4,2	4,3
Game 1		-1	-1	1		1	1	-1		-1	-1	-1		1	1	-1
Game 2		-1	-1	1		-1	1	1		-1	-1	-1		-1	1	-1
Game 3		-1	1	-1		1	-1	1		-1	-1	-1		-1	-1	-1
Game 4		-1	-1	-1		1	1	1		1	1	-1		1	-1	-1
Game 5		-1	-1	-1		1	1	1		-1	-1	-1		-1	1	-1
Game 6		-1	-1	1		-1	-1	1		-1	-1	-1		1	1	-1
Game 7		1	-1	1		1	1	1		-1	-1	-1		1	1	1
Game 8		1	1	1		1	1	1		-1	-1	-1		-1	-1	-1
Game 9		1	-1	1		1	1	1		1	1	-1		1	1	-1
Game 10		-1	1	-1		1	-1	1		1	-1	-1		-1	1	-1

The table below, based on the results of the above tabulation, shows the effectiveness of each strategy. A negative result (represented with red) shows that a particular

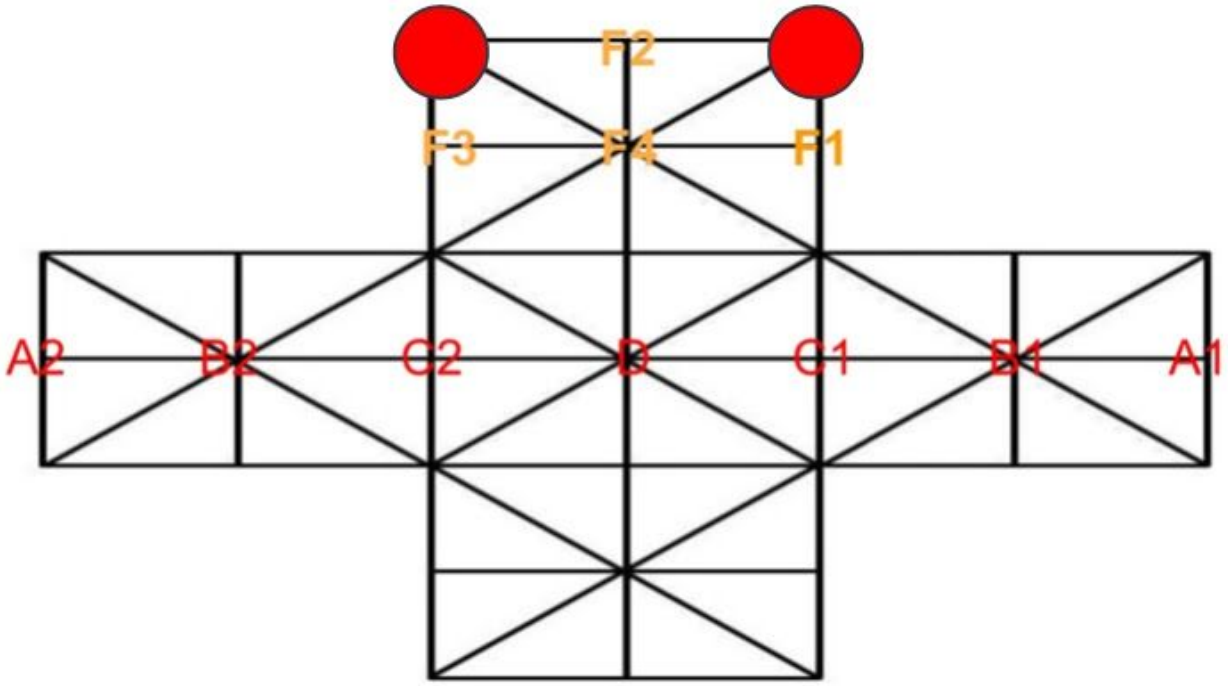
strategy is generally ineffective. A positive result (represented with green) shows that a particular strategy is generally effective. The bigger the number, the more ineffective or effective the strategy is.

EFFECTIVENESS		
-0.6	SHEEP 1	Trap fox by sacrificing sheep
1.8	SHEEP 2	Have sheep stay as close as possible
-2.0	SHEEP 3	Have sheep get to winning space as fast as possible
-0.4	SHEEP 4	Sacrifice sheep to get fox into a bad position, then get sheep into winning space
0.2	FOX 1	Have 1 fox in winning area to prevent sheep from reaching winning area
0.2	FOX 2	Corner multiple sheep
0.8	FOX 3	Allow sheep to get to winning area, so that sheep is spread out. Then, kill as many sheep as possible.

Unsurprisingly, Sheep 3 seems totally powerless to the fox strategies, while all the fox strategies are powerless to Sheep 2. Since Sheep 2 is so effective, we have decided to further investigate this strategy.

Further investigation of Sheep 2

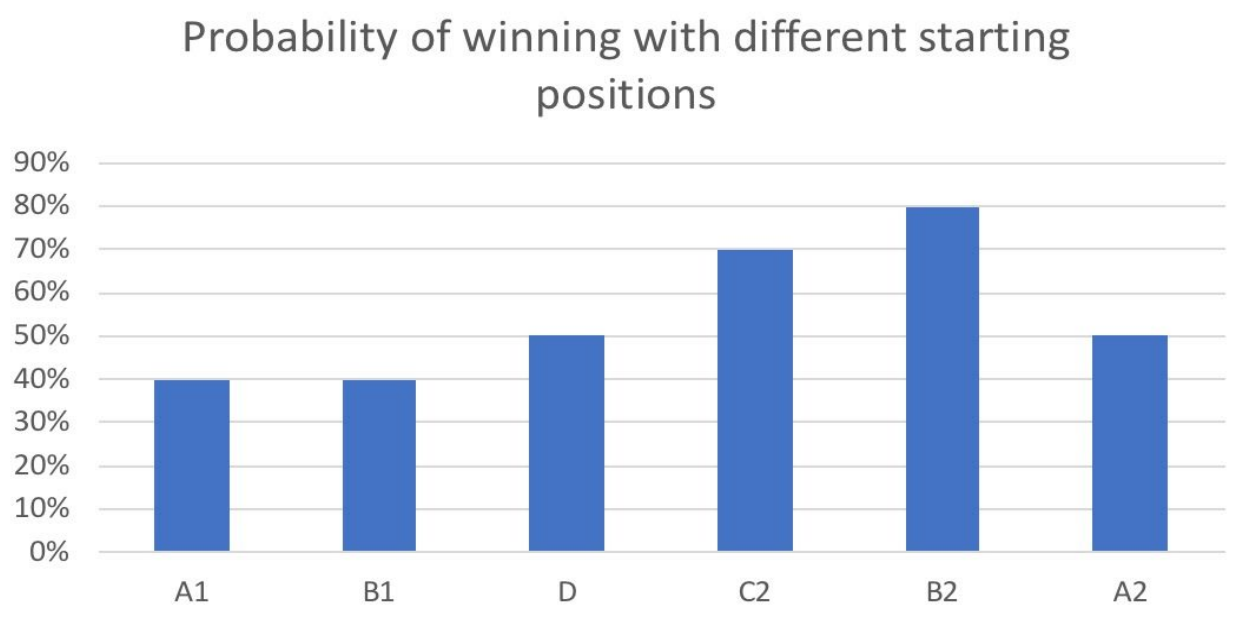
It can be seen that sheep strategy 2 is very effective, but to what certain extent? Only when a player masters the strategy. One of the ways of mastering this strategy is to decide on your first move as a sheep player, as a wrong starting move could see you at a disadvantage. Hence, we have investigated too what is the best starting position for the sheep. This, however is situational, as the fox player starts first, and he/she can move differently. Below, is the position of certain sheep and fox. Yellow indicates the positions that either fox can move to on its first turn, while red indicates the starting position of sheep. Red circles refer to the original starting positions for the foxes.


F2
F3
F4
F1
A2
B2
C2
D
C1
B1
A1

Case 1: Fox moves to F1

The table shows the results of different starting positions from sheep. Note that the sheep from C1 cannot be moved, as that would give the fox a free meal.

If fox move to F1	A1	B1	D	C2	B2	A2
Game 1	Lose	Lose	Lose	Lose	59	Lose
Game 2	Lose	34	71	52	Lose	Lose
Game 3	87	37	36	49	48	44
Game 4	46	53	Lose	26	41	59
Game 5	Lose	Lose	69	Lose	50	Lose
Game 6	Lose	Lose	40	37	Lose	64
Game 7	Lose	Lose	Lose	48	71	68
Game 8	66	Lose	57	52	33	Lose
Game 9	55	Lose	Lose	Lose	69	Lose
Game 10	Lose	42	Lose	60	33	99
Average	63.5	41.5	54.6	46.2857	50.5	66.8



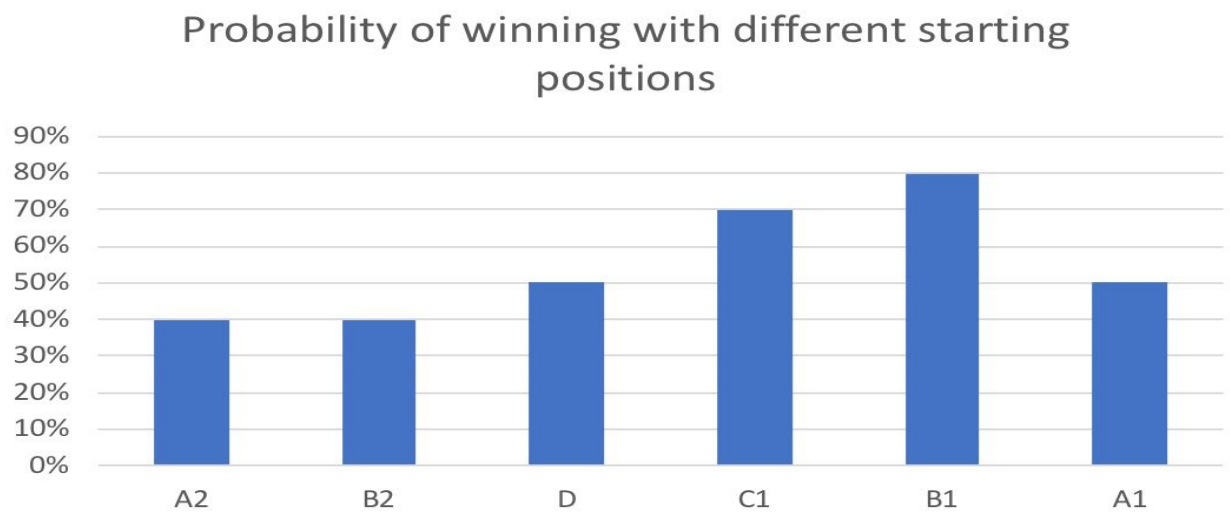
Hence, Moving the sheep from B2 is most suitable.

Case 2: Fox moves to F2

This is an unlikely situation, however, as such a move would allow the sheep player to move any sheep ahead without traps, and the fox will get trapped faster. Hence, we have avoided this situation as in such a case, the fox will lose almost all the time.

Case 3: Fox moves to F3

Due to symmetry, moving A1 gives the same results as A2 in case 1. Moving B1 gives the same results as B2 in case 1, and D will give the same results as D in case 1, and vice versa. Moving C1 gives the same results as C2 in case 1, but not the other way round as moving C2 will get the sheep eaten.



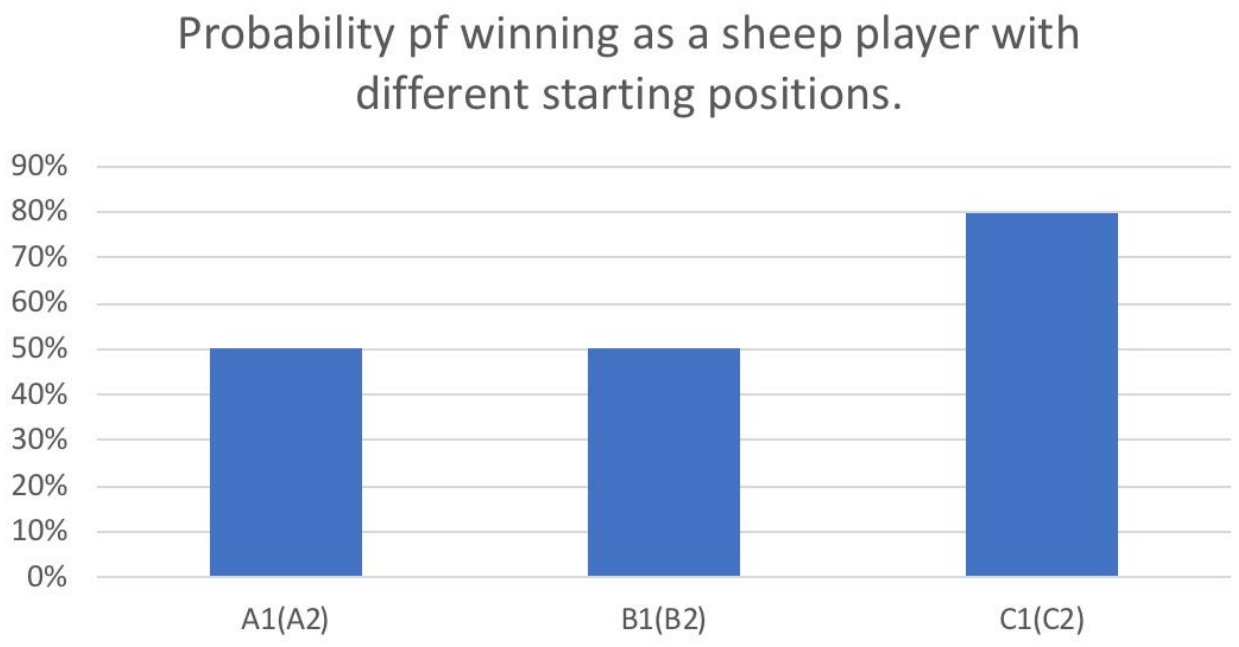
Hence, moving B1 will be most suitable.

Case 4: Fox moves to F4

This is the most probable situation. The table below shows the results of moving sheep from different positions in such a situation. Due to symmetry , A1 gives the same results as A2, as do B1 and B2, C1 and C2. Note that the sheep from D cannot be moved, as it would give the foxes a free meal.

If fox move to F4	A1(A2)	B1(B2)	C1(C2)
Game 1	Lose	Lose	66
Game 2	55	68	52
Game 3	46	63	60
Game 4	Lose	Lose	71
Game 5	Lose	Lose	55
Game 6	Lose	41	47
Game 7	65	68	45
Game 8	Lose	Lose	Lose
Game 9	59	Lose	45
Game 10	39	59	Lose
Average	52.8	59.8	55.125

And hence, the probability of winning with different starting positions.



From the above results, we can conclude that moving the sheep from C1 or C2 is the best option in Case 4.

4.2.2 Research Question 2: How does the number of foxes and sheep affect gameplay?

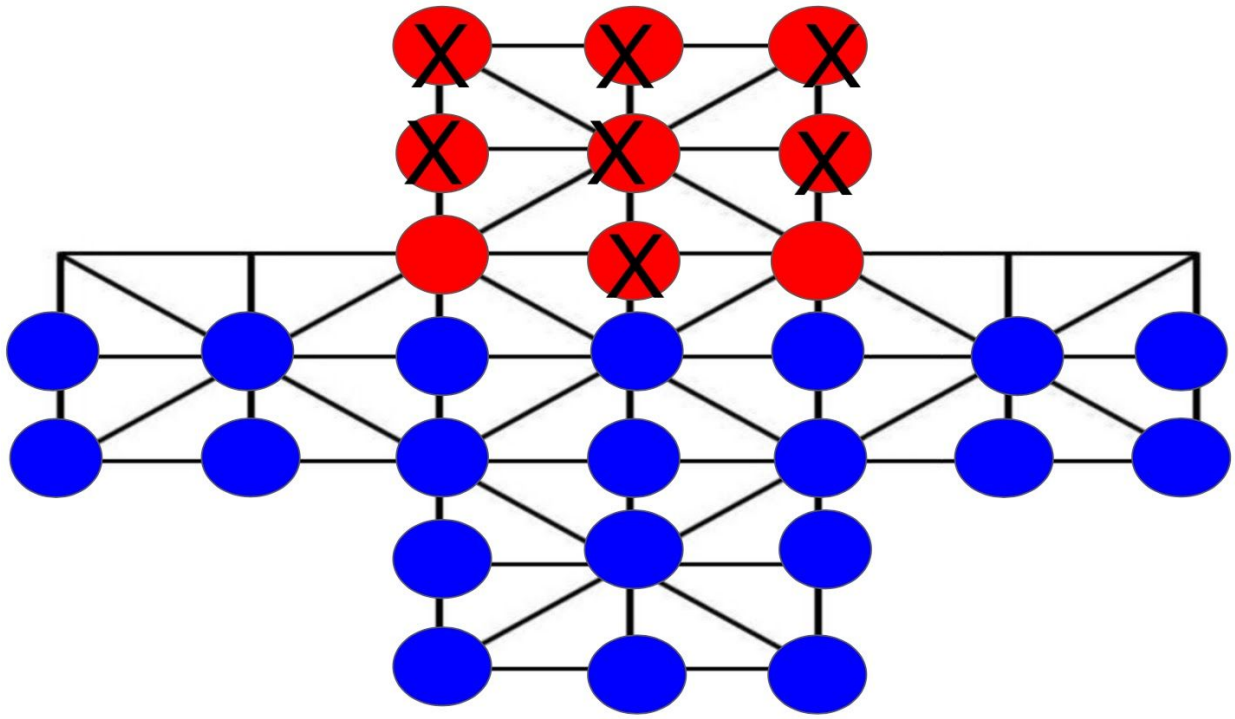
Fox

For the number of foxes, we have tabulated by using Fox 3 (the most effective fox strategy) for all the different numbers of foxes, ranging from 1 to 4 foxes. The results are as follows.

Fox 3	2 foxes	3 foxes	4 foxes	1 fox
Game 1	Lose	31	25	Lose
Game 2	Lose	60	25	Lose
Game 3	Lose	Lose	Lose	Lose
Game 4	Lose	60	81	Lose
Game 5	Lose	55	Lose	Lose
Game 6	76	52	74	48
Game 7	73	Lose	43	Lose
Game 8	43	67	39	Lose
Game 9	43	62	43	Lose
Game 10	51	59	63	Lose
Game 11	Lose	57	44	Lose
Average	57.2	55.88888889	48.55555556	48

Rather unsurprisingly, as the number of foxes increases from 2 to 3, the occurrence of a fox winning increases. When we decrease that number from 2 to 1, the occurrence of a fox winning decreases significantly.

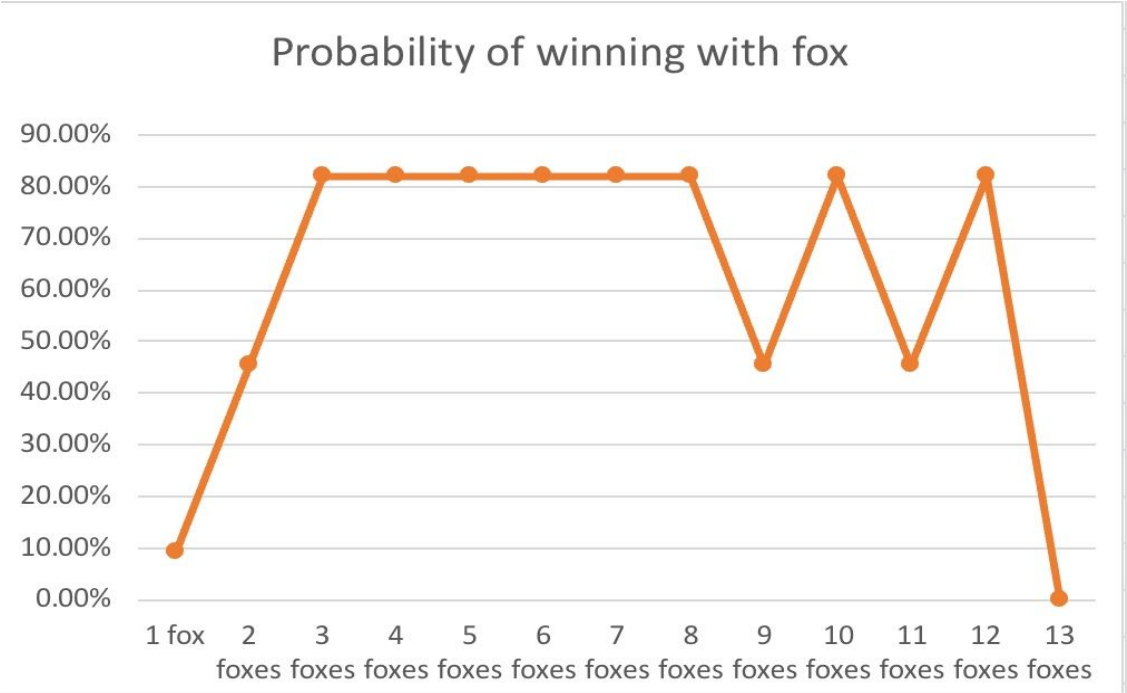
At 4 foxes, the occurrence of winning starts to become constant. Hence, it can be inferred that more than 4 foxes would still give similar results. This is because at 10 foxes, for example, some foxes will be removed from gameplay before the first turn, and will eventually reduce to 3 or 4 foxes. Two exceptions, however, is when there are 11 or 9 foxes, as the number of foxes will be reduced to 2 before the first turn.



This is what happens with 9 foxes. As seen in the diagram, 7 foxes are eliminated before the first turn, marked with a black cross.

At 13 foxes, the foxes will lose immediately, as all foxes have nowhere to go. Hence, all foxes will be removed from gameplay even before the first turn.

When there are more than 13 foxes, the game is impossible to play.



Hence, the probability of winning with different numbers of foxes, from 1 to 13.

Sheep

As for the number of sheep, we have tabulated using Sheep 2 (the most effective sheep strategy) for all different numbers of sheep, ranging from 13 to 27 . The results are as follows.

Strategy 2 (sheep)	20 sheep	13 sheep	17 sheep	22 sheep	27 sheep
Game 1	46	Lose	Lose	47	27
Game 2	64	Lose	Lose	38	43
Game 3	98	Lose	Lose	62	23
Game 4	66	Lose	68	39	37
Game 5	52	Lose	Lose	32	27
Game 6	60	Lose	Lose	25	44
Game 7	71	Lose	Lose	22	56
Game 8	55	Lose	Lose	29	19
Game 9	Lose	Lose	Lose	58	31
Game 10	47	Lose	70	60	26
Game 11	Lose	Lose	Lose	42	22
Average	62.11111111	NIL	69	41.27272727	32.27272727

And hence, the probability of winning as a sheep player with different numbers of sheep.

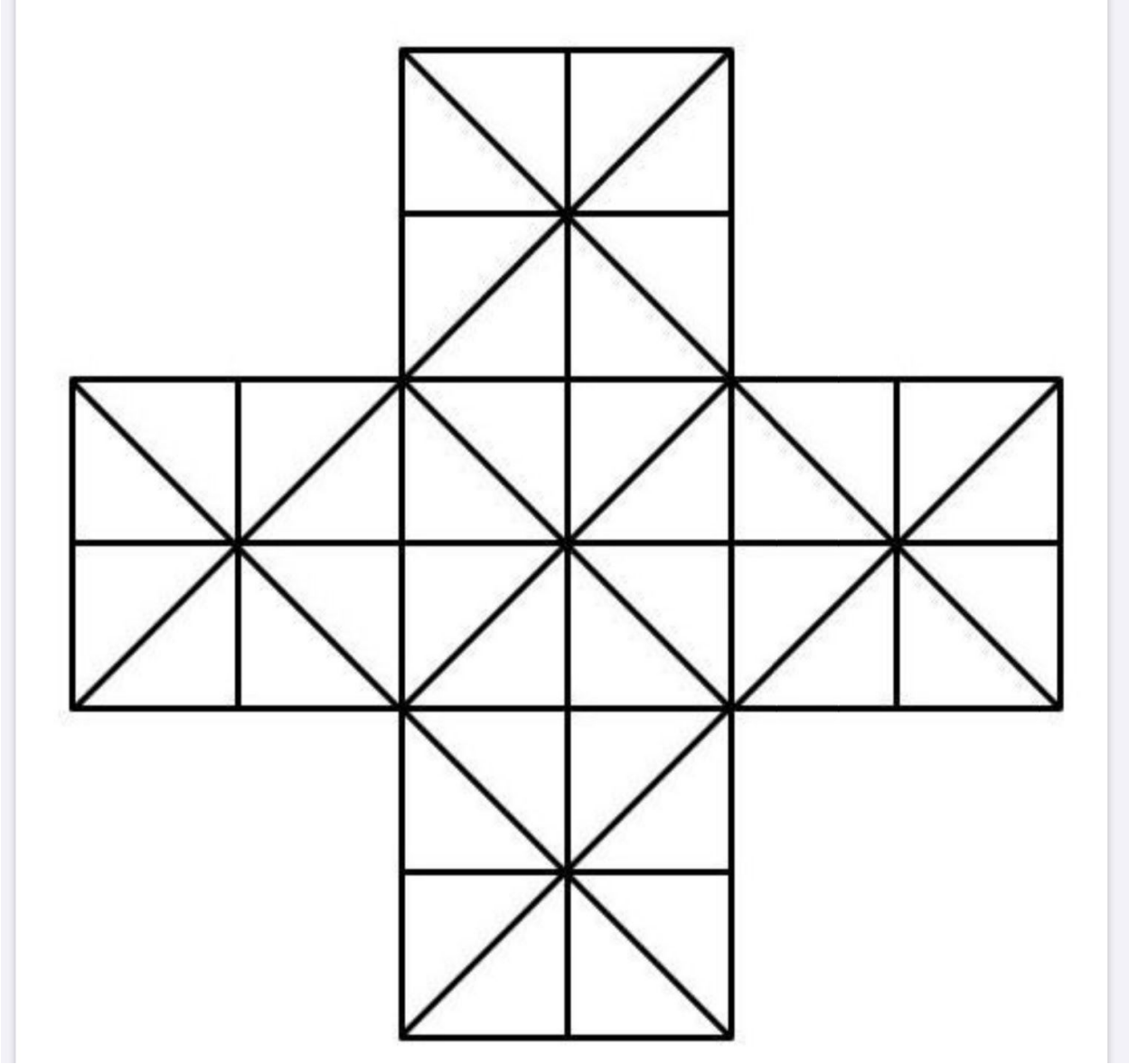
As seen from the results, reducing the number of sheep will cause the occurrence of sheep winning to decrease drastically, while increasing the number of sheep will significantly increase the occurrence of the sheep winning. The number of moves taken for the sheep to win also differs, with that number decreasing when there are more sheep, and the same number of moves taken increasing when the number of sheep decreases.

The fact that there is a chance of winning as the fox player at 20 sheep when Sheep 2 (the best strategy in the game) is used, shows that at 20 sheep (at least, generally), the fox player has a relatively fair chance of winning. (That is, if the sheep player decides to use a random strategy.)

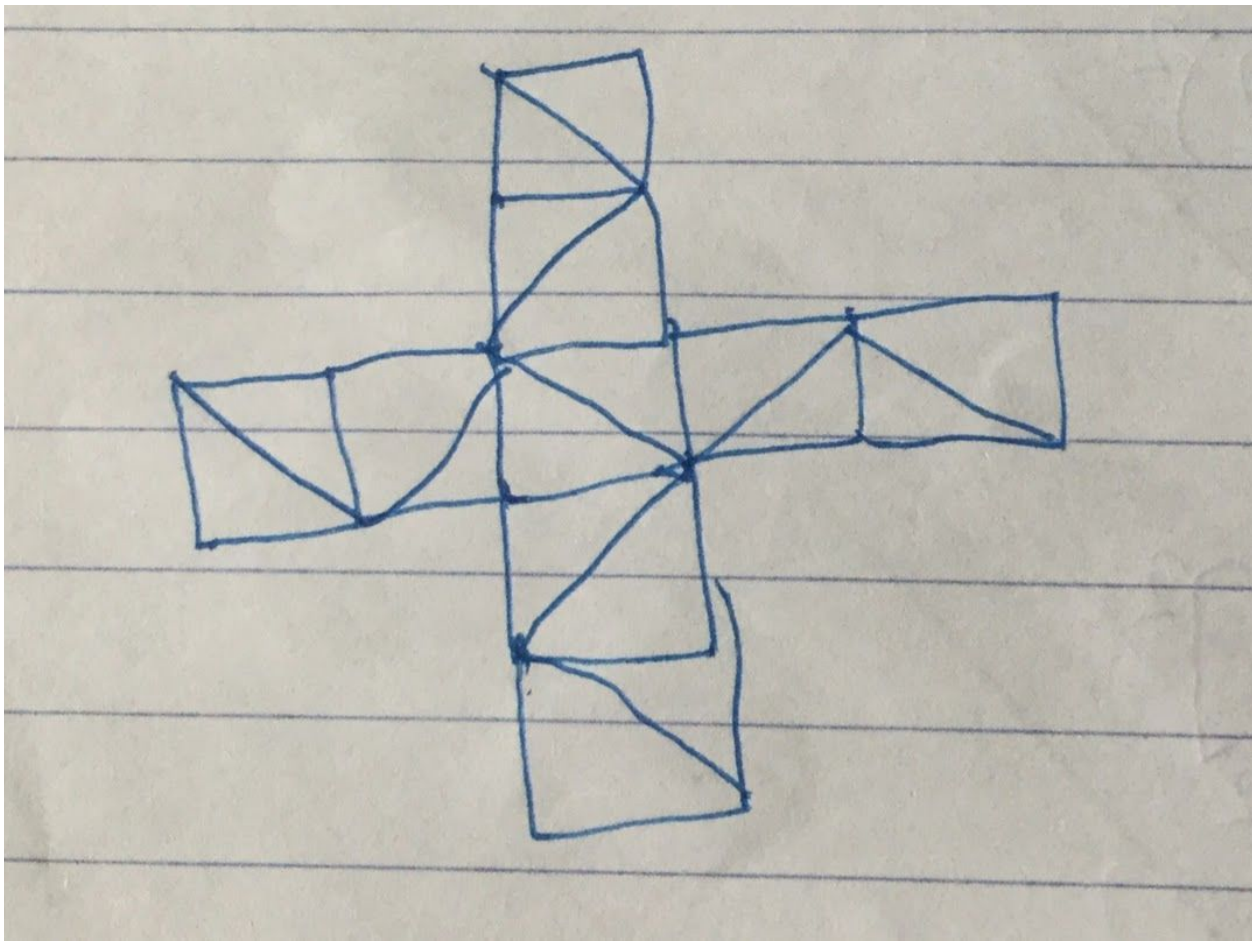
All in all, it can be explained why there are 2 foxes and 20 sheep, as a different number would cause an imbalance to the game, whereby the fox or the sheep wins too much.

4.2.3 Research Question 3. How does the dimensions of the board affect the game?

This, however, will make the game imbalanced. The original board is in 3-by-3 dimensions, in the shape of a plus '+'. There are 33 spaces, as shown below.

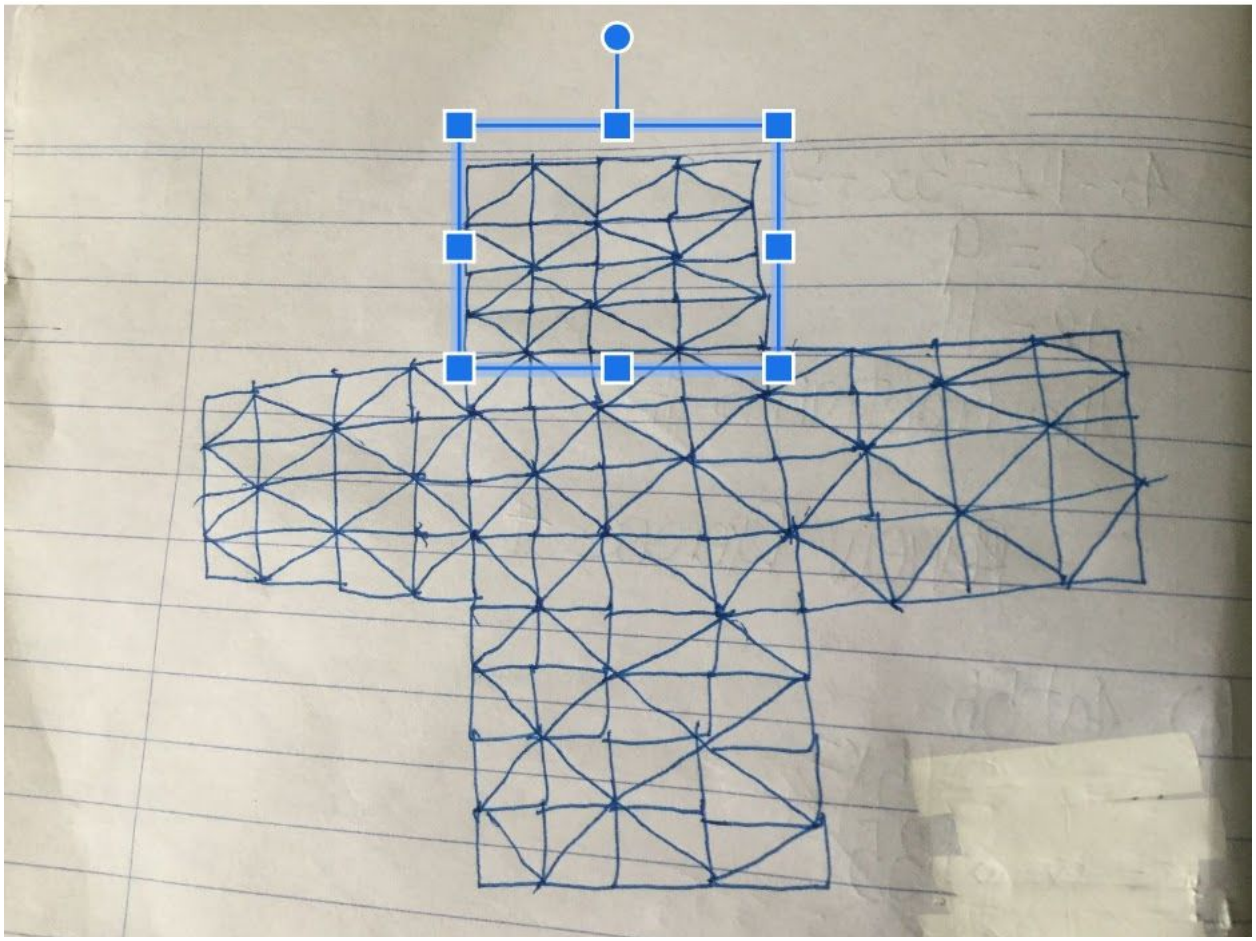


If we were to reduce that to a 2-by-2 plus, there will be only 20 spaces left, as shown below, insufficient for 20 sheep and 2 fox. Thus, the size cannot be reduced, as the game will not be playable.



Secondly, the board is in a n-by-n plus ‘+’. If n is even, the board will have no centre, as seen above,thus the board is not symmetrical, making the game unfair. So, n has to be odd.

Then, suppose we expand the board to a 5-by-5 plus ‘+’, because we have only 20 sheep, it is not enough to fully occupy the other side (shown in the blue box, which has 25 spaces). Thus, the game is not playable. Needless to say, anything bigger gives the same results.



Thus, with the same number of sheep and fox, the only playable dimension is 3-by-3.

Hence, it can also be explained why the board is in a 3-by-3 plus '+', and not in other dimensions.

(5) **Possible extensions from this project**

1. Find out more ways to master Sheep 2.
2. Find out what happens if we change the shape of the board (i.e. from a plus to a square or a triangle)

(6) **Reflections**

Elliot (Leader):

As a group leader, no doubt I felt some pressure, especially since there is COVID-19. As we were unable to physically meet during the holidays, we were delayed in our tabulations (thankfully PW will not be counted towards EOY MSG if we don't do well). Hence, I am glad we were able to finish this project, although there is room for improvement. Take time management for example. I've heard that many Sec 1s in HCI (not just 2020 Sec 1s, but all Sec 1s from different cohorts) had trouble with their time management in the process of doing their projects, and I'm afraid to say that our group is no exception. We started the tabulation quite late, as a result we had to rush a bit towards the end. If I could turn back the time, I would start earlier, but I'm afraid that's impossible. So it would be time for me to turn the page, carry along the lessons I've learnt from this project, and do better next year.

Xizhe (Member):

This is surely an interesting and enjoyable project, where I had fun and worked well with my teammates. When I first played the game, I thought it was very unfair for the sheep, because I thought that the fox is overpowered. However, as I played the game more times, I discovered many new moves by the sheep that can easily counter the fox. This made the process fun, especially when I made a comeback with an unexpected move and turned the game around. This made me realise that in life, we should never give up, because there is always hope. COVID-19 had certainly affected our tabulation sessions, which left us with little time so we had to speed up. But overall, I think we did our best, even though there is still room for improvement, and this is what I think matters.

Pei You (Member):

This project has certainly helped me learn how to adapt to a different and unprecedented event such as COVID-19. The virus has caused us a lot of stress and strain as our tabulation sessions all had to be delayed, causing us to be on a very tight schedule. However, this project has been a very interesting one and the blood, sweat and tears my teammates and I have put into it are certainly worth it based on the end

product. This project has been a very fun and interesting one, and a follow-up project might be possible.

Ryan (Member):

In this project, we have done many things, in a short period of time. Due to COVID-19, we were unable to tabulate during the circuit breaker, but we overcame all odds and finished all that we had to do on time. I have really enjoyed this project, doing research on the fox and sheep game. In fact, I have never really embarked on such a large-scale project before. I have taken numerous things away from this project, such as working together as a team. Next time, I would do my best to ensure the team practices time management better. All in all, I would love to embark on similar projects soon.

(7) **References**

History:
https://en.wikipedia.org/wiki/Fox_games#History Retrieved on 14 March 2020
http://www.cyningstan.com/game/57/fox-geese Retrieved on 13 March 2020

Do note that these websites contain different variations of the game, so do not get confused.

Images:
https://www.whatdowedoallday.com/wp-content/uploads/2017/02/fox_and_geese1-2.pdf
Retrieved on 29 March 2020

Other images are original.