SONA

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Class:

1A3

Contents

page

1. INTRODUCTION

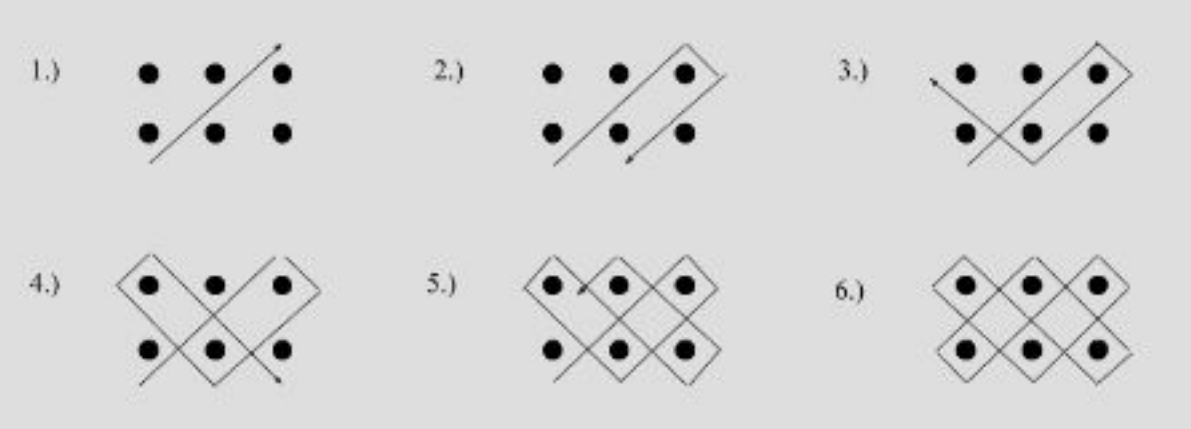
1.1 WHAT ARE SONA

Sona, lusona as singular, are ancient drawings that are formed by lines and dots. They are simple to draw and are usually symmetrical.

1.1.1 HISTORY OF SONA

Sona have been around since 300 years ago. It originated from the Chokwe people of Angola. It is used to tell stories and legends. The storyteller will draw the diagrams on sand with a wooden stick.

1.1.2 HOW TO DRAW A LUSONA



Start the line from in between two dots. Extend the line at a 45-degree angle. When outside the dot array, turn 90-degrees, then extend the line until it is outside the dot array again. Continue to do this until the line reaches its starting point.

Repeat this at a different starting point until all the dots have been enclosed.

1.2 RATIONALE

Since sona are very interesting drawings, we are passionate to learn more explore about sona's relation to mathematics and how mathematics may be applied to figure out patterns in rectangular sona configurations.

1.3 AIM

The aim of our project is to find out how many unbroken lines, intersection points, and faces there are in a lusona of a rectangular array of **a** x **b,** where $\mathbf{a} \leq \mathbf{b} \leq 10$ and $\mathbf{a, b} \in \mathbb{N}$. Besides, we want to find a formula which derives the number of unbroken lines, intersection points, and faces in similar lusona from the objective.

1.4 OBJECTIVES

Our three objectives are as follows;

A. To find the amount of intersection points, faces and unbroken lines there are in a lusona of a rectangular array of **a** x **b,** where $\mathbf{a} \leq \mathbf{b} \leq 10$ and $\mathbf{a, b} \in \mathbb{N}$.

B. To deduce formulae to calculate the amount of intersection points, faces and unbroken lines there are in a lusona of a rectangular array of **a** x **b,** where $\mathbf{a} \leq \mathbf{b} \leq 10$ and $\mathbf{a, b} \in \mathbb{N}$.

C. To prove the formulae in Objective B.

1.5 FIELD OF MATHS

Algebra and simple arithmetic have been used to achieve our objectives.

2. LITERATURE REVIEW

Paul Ellis of Manhattanville College had written a study on sona titled “The mathematics of Sona”. He showed that there are many different types of sona diagrams, and that there could be many more other configurations.

3. TERMINOLOGY

Unbroken line - a line that ultimately returns to its starting point after going around the lusona.

Faces - individual areas separated by the unbroken lines in the lusona.

Intersection point - the point of which an unbroken line intersects another or itself.

4. STUDY AND METHODOLOGY

Firstly, some literature review was done. Secondly, all of the **a x b** rectangular array where **a ≤ b ≤ 10** of sona was drawn. Next, patterns were observed from the data found. Then, formulae were made and proven. Lastly, the research questions were solved.

4.1 RESEARCH QUESTIONS

Our three research questions are as follows;

A. How many intersection points, faces and unbroken lines are there in a lusona of a rectangular array of **a** x **b,** where **a** ≤ **b** ≤ 10 and **a, b** ∈ ℕ?

B. What are the formulae to calculate the amount of intersection points, faces and unbroken lines there are in a lusona of a rectangular array of **a** x **b,** where **a** ≤ **b** ≤ 10 and **a, b** ∈ ℕ?

C. How do we prove the formulae in Research Question B?

4.2 INVESTIGATION AND DATA

Sona of an **a** x **b** rectangular array where **a ≤ b ≤ 10** were drawn.

The picture below shows an example of the sona that were drawn.

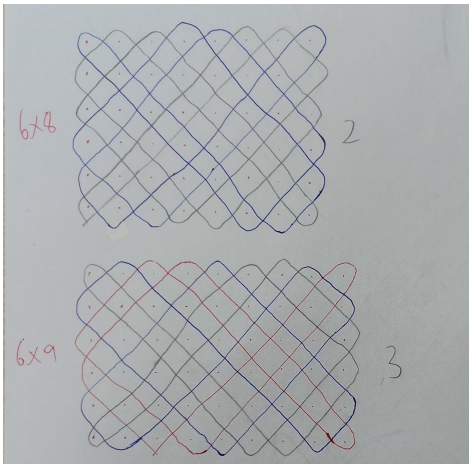


Figure 1

We did observations on the amount of faces, intersections and unbroken lines were tabulated.

Rows (a)	Factors of a	Columns (b)	Factors of b	Common factors of a & b	HCF of a & b	No. of Unbroken lines	No. of Intersect -ions	No. of Faces
1	1	1	1	1	1	1	0	1
1	1	2	1, 2	1	1	1	1	2
1	1	3	1, 3	1	1	1	2	3
1	1	4	1, 2, 4	1	1	1	3	4
1	1	5	1, 5	1	1	1	4	5
1	1	6	1, 2, 3, 6	1	1	1	5	6
1	1	7	1, 7	1	1	1	6	7
1	1	8	1, 2, 4, 8	1	1	1	7	8
1	1	9	1, 3, 9	1	1	1	8	9
1	1	10	1, 2, 5, 10	1	1	1	9	10
2	1, 2	2	1, 2	1, 2	2	2	4	5
2	1, 2	3	1, 3	1	1	1	7	8
2	1, 2	4	1, 2, 4	1, 2	2	2	10	11
2	1, 2	5	1, 5	1	1	1	13	14
2	1, 2	6	1, 2, 3, 6	1, 2	2	2	16	17
2	1, 2	7	1, 7	1	1	1	19	20
2	1, 2	8	1, 2, 4, 8	1, 2	2	2	22	23
2	1, 2	9	1, 3, 9	1	1	1	25	26
2	1, 2	10	1, 2, 5, 10	1, 2	2	2	28	29

Rows (a)	Factors of a	Columns (b)	Factors of b	Common factors of a & b	HCF of a & b	No. of Unbroken lines	No. of Intersect -ions	No. of Faces
3	1, 3	3	1, 3	1, 3	3	3	12	13
3	1, 3	4	1, 2, 4	1	1	1	17	18
3	1, 3	5	1, 5	1	1	1	22	23
3	1, 3	6	1, 2, 3, 6	1, 3	3	3	27	28
3	1, 3	7	1, 7	1	1	1	32	33
3	1, 3	8	1, 2, 4, 8	1	1	1	37	38
3	1, 3	9	1, 3, 9	1, 3	3	3	42	43
3	1, 3	10	1, 2, 5, 10	1	1	1	47	48
4	1, 2, 4	4	1, 2, 4	1, 2, 4	4	4	24	25
4	1, 2, 4	5	1, 5	1	1	1	31	32
4	1, 2, 4	6	1, 2, 3, 6	1, 2	2	2	38	39
4	1, 2, 4	7	1, 7	1	1	1	45	46
4	1, 2, 4	8	1, 2, 4, 8	1, 2, 4	4	4	52	53
4	1, 2, 4	9	1, 3, 9	1	1	1	59	60
4	1, 2, 4	10	1, 2, 5, 10	1, 2	2	2	66	67
5	1, 5	5	1, 5	1, 5	5	5	40	41
5	1, 5	6	1, 2, 3, 6	1	1	1	49	50
5	1, 5	7	1, 7	1	1	1	58	59
5	1, 5	8	1, 2, 4, 8	1	1	1	67	68

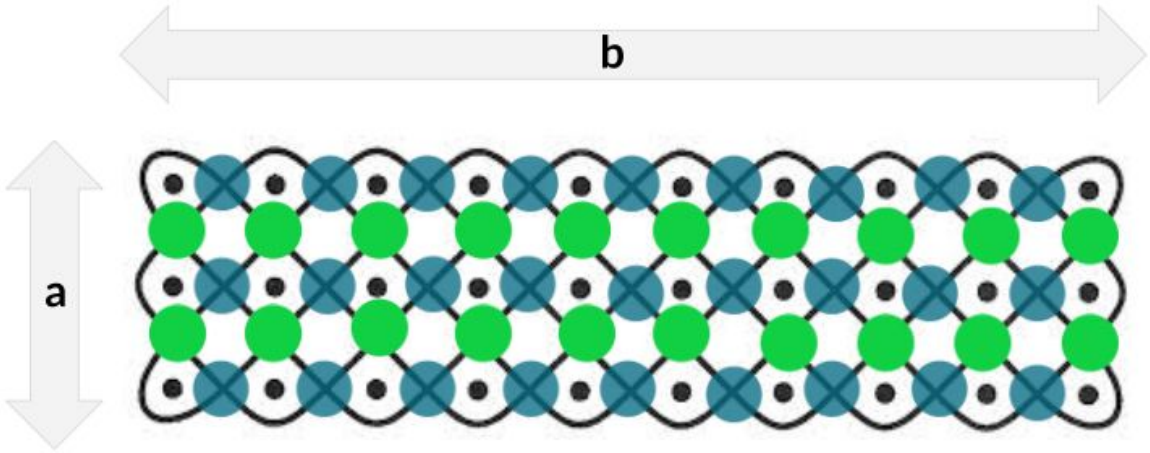
Rows (a)	Factors of a	Columns (b)	Factors of b	Common factors of a & b	HCF of a & b	No. of Unbroken lines	No. of Intersect -ions	No. of Faces
5	1, 5	9	1, 3, 9	1	1	1	76	77
5	1, 5	10	1, 2, 5, 10	1,5	5	5	85	86
6	1, 2, 3, 6	6	1, 2, 3, 6	1, 2, 3, 6	6	6	60	61
6	1, 2, 3, 6	7	1, 7	1	1	1	71	72
6	1, 2, 3, 6	8	1, 2, 4, 8	1, 2	2	2	82	83
6	1, 2, 3, 6	9	1, 3, 9	1, 3	3	3	93	94
6	1, 2, 3, 6	10	1, 2, 5, 10	1,2	2	2	104	105
7	1, 7	7	1, 7	1, 7	7	7	84	85
7	1, 7	8	1, 2, 4, 8	1	1	1	97	98
7	1, 7	9	1, 3, 9	1	1	1	110	111
7	1, 7	10	1, 2, 5, 10	1	1	1	123	124
8	1, 2, 4, 8	8	1, 2, 4, 8	1, 2, 4, 8	8	8	112	113
8	1, 2, 4, 8	9	1, 3, 9	1	1	1	127	128
8	1, 2, 4, 8	10	1, 2, 5, 10	1, 2	2	2	142	143
9	1, 3, 9	9	1, 3, 9	1, 3, 9	9	9	144	145
9	1, 3, 9	10	1, 2, 5, 10	1	1	1	161	162
10	1, 2, 5, 10	10	1, 2, 5, 10	1, 2, 5, 10	10	10	180	181

4.3 FORMULAE AND PROOFS

Algebra and observation was applied to prove the formulae deduced from the data.

4.3.1 AMOUNT OF INTERSECTION POINTS

Rows (a)	Columns (b)	No. of Intersect -ions	Rows (a)	Columns (b)	No. of Intersect -ions	Rows (a)	Columns (b)	No. of Intersect -ions
1	1	0	3	3	12	5	9	76
1	2	1	3	4	17	5	10	85
1	3	2	3	5	22	6	6	60
1	4	3	3	6	27	6	7	71
1	5	4	3	7	32	6	8	82
1	6	5	3	8	37	6	9	93
1	7	6	3	9	42	6	10	104
1	8	7	3	10	47	7	7	84
1	9	8	4	4	24	7	8	97
1	10	9	4	5	31	7	9	110
2	2	4	4	6	38	7	10	123
2	3	7	4	7	45	8	8	112
2	4	10	4	8	52	8	9	127
2	5	13	4	9	59	8	10	142
2	6	16	4	10	66	9	9	144
2	7	19	5	5	40	9	10	161
2	8	22	5	6	49	10	10	180
2	9	25	5	7	58	The formula is **2ab - a - b**		
2	10	28	5	8	67			



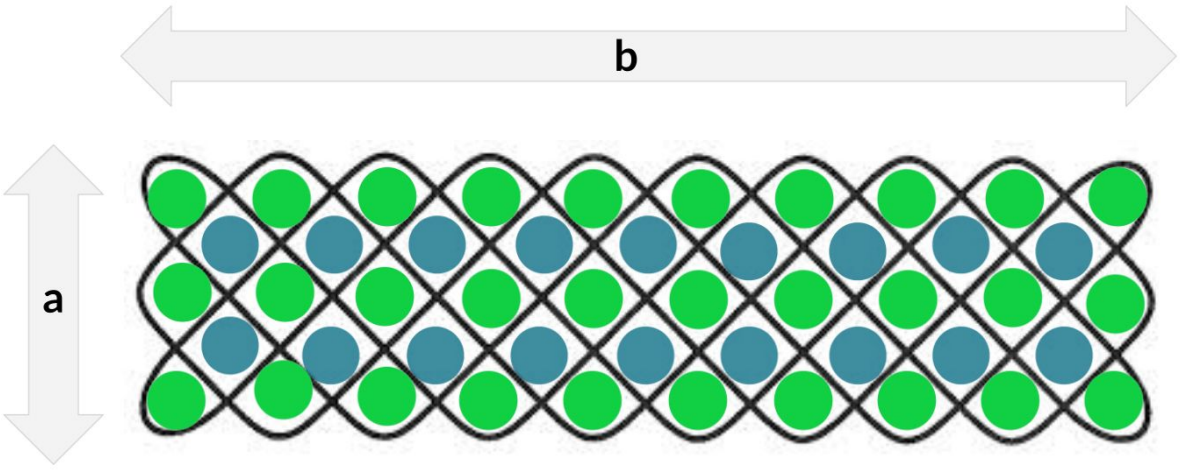
Number of intersections marked green
= **b** (**a** - 1)
= **ab - b**

Number of intersections marked blue
= **a** (**b** - 1)
= **ab - a**

Number of intersections in total
= (**ab - b**) + (**ab - a**)
= **ab - b** + **ab - b**
= 2**ab - a - b**

4.3.2 AMOUNT OF FACES

Rows (a)	Columns (b)	No. of Faces	Rows (a)	Columns (b)	No. of Faces	Rows (a)	Columns (b)	No. of Faces
1	1	1	3	3	13	5	9	77
1	2	2	3	4	18	5	10	86
1	3	3	3	5	23	6	6	61
1	4	4	3	6	28	6	7	72
1	5	5	3	7	33	6	8	83
1	6	6	3	8	38	6	9	94
1	7	7	3	9	43	6	10	105
1	8	8	3	10	48	7	7	85
1	9	9	4	4	25	7	8	98
1	10	10	4	5	32	7	9	111
2	2	5	4	6	39	7	10	124
2	3	8	4	7	46	8	8	113
2	4	11	4	8	53	8	9	128
2	5	14	4	9	60	8	10	143
2	6	17	4	10	67	9	9	145
2	7	20	5	5	41	9	10	162
2	8	23	5	6	50	10	10	181
2	9	26	5	7	59	The formula is $2\mathbf{ab} - \mathbf{a} - \mathbf{b} + 1$		
2	10	29	5	8	68			



Number of faces marked green
= **ab**

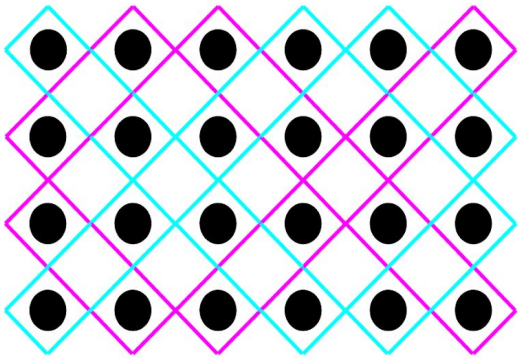
Number of faces marked blue
= (**a** - 1) (**b** - 1)
= **ab** - **a** - **b** + 1

Number of faces in total
= **ab** + (**ab** - **a** - **b** + 1)
= **ab** + **ab** - **a** - **b** + 1
= 2**ab** - **a** - **b** + 1

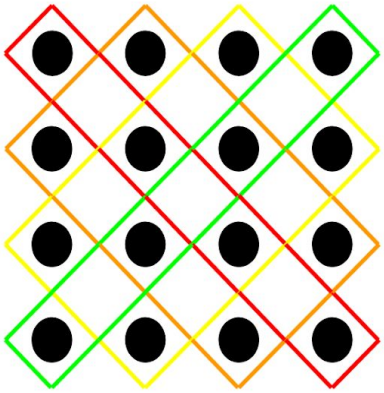
4.3.3 AMOUNT OF UNBROKEN LINES

Rows (a)	Columns (b)	No. of Unbroken lines	Rows (a)	Columns (b)	No. of Unbroken lines	Rows (a)	Columns (b)	No. of Unbroken lines
1	1	1	3	3	3	5	9	1
1	2	1	3	4	1	5	10	5
1	3	1	3	5	1	6	6	6
1	4	1	3	6	3	6	7	1
1	5	1	3	7	1	6	8	2
1	6	1	3	8	1	6	9	3
1	7	1	3	9	3	6	10	2
1	8	1	3	10	1	7	7	7
1	9	1	4	4	4	7	8	1
1	10	1	4	5	1	7	9	1
2	2	2	4	6	2	7	10	1
2	3	1	4	7	1	8	8	8
2	4	2	4	8	4	8	9	1
2	5	1	4	9	1	8	10	2
2	6	2	4	10	2	9	9	9
2	7	1	5	5	5	9	10	1
2	8	2	5	6	1	10	10	10
2	9	1	5	7	1	The formula is HCF (**a , b**)		
2	10	2	5	8	1			

Colour each unbroken line a different colour.



For sona on a square display,



For any square lusona, there is always two unbroken lines which take only two dots from the outer layer*. The other unbroken lines take four dots from the outer layer*.

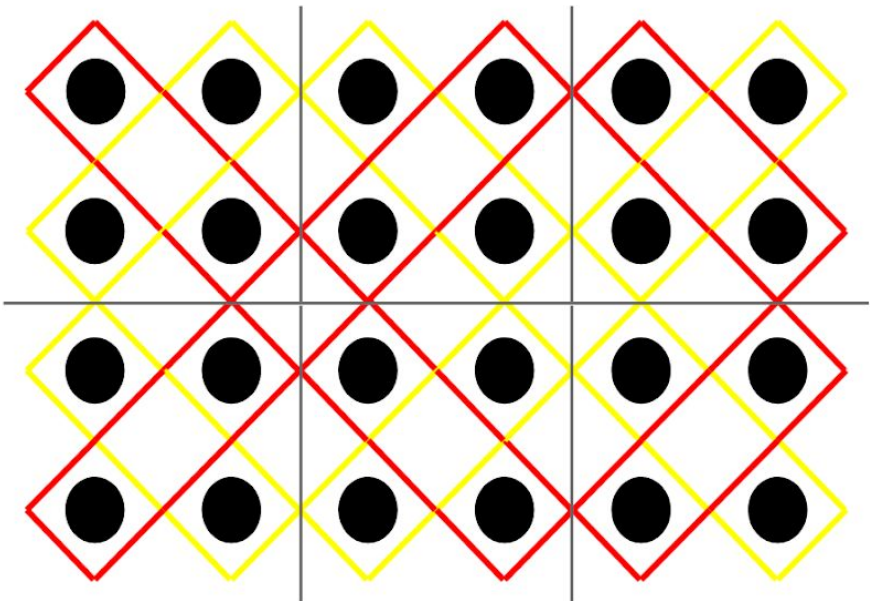
Also, for any square lusona of a dot array of **n** x **n**, there are 4 (**n** -1) dots.

Let the amount of unbroken lines be **y**.
2 x 2 + 4 (**y** - 2) = 4 (**n** - 1)
4 + 4**y** - 8 = 4**n** - 4
y = **n**

This means that the amount of unbroken lines is always the width of the square lusona, which is also the HCF(width, length), because the length is equal to the width.

*dots on outer layer means those that have less than 4 adjacent dots.

For sona on a rectangular display, first draw lines that may be called “mirror lines”.



Each part of the lusona separated by the mirror lines are a reflection of the part adjacent to it, even by colour!

Each segment is also a square, which has the dimensions of HCF(width, length)!

This means that any rectangular lusona can be formed by “reflecting” the square lusona with the dimensions of HCF(width, length)!

Thus it has the same amount of unbroken lines as the aforementioned square lusona!

5. CONCLUSION

5.1 CONCLUSION OF RESEARCH QUESTION A

We have found the amount of intersection points, faces and unbroken lines there are in a lusona of a rectangular array of **a** x **b,** where $\mathbf{a} \leq \mathbf{b} \leq 10$ and $\mathbf{a, b} \in \mathbb{N}$.

Results are as shown in the *Investigations and Data* section.

5.2 CONCLUSION OF RESEARCH QUESTION B

We have found the formulae for the amount of intersection points, faces and unbroken lines there are in a lusona of a rectangular array of **a** x **b,** where $\mathbf{a} \leq \mathbf{b} \leq 10$ and $\mathbf{a, b} \in \mathbb{N}$.

Amount of item	***Formula***
Amount of intersection points	$2\mathbf{ab} - \mathbf{a} - \mathbf{b}$
Amount of faces	$2\mathbf{ab} - \mathbf{a} - \mathbf{b} + 1$
Amount of unbroken lines	$\text{HCF}(\mathbf{a}, \mathbf{b})$

5.3 CONCLUSION OF RESEARCH QUESTION C

Proofs for the formulae in research question 2 is completed.

Results are as shown in the *Formulae and Proofs* section.

6. REFERENCES

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