Cat 9 - Mathematics Research Report

Win, or lose?

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Contents

1. Abstract:

This project is a study about the application and extension of some fallacies in gambling. In our research, we stand in gambler's shoes to explain why so many are keen to gamble and why they are doom to lose money. Also, we stand in the casino's shoes to explain how casinos set traps for gamblers in gambling games. We hope to use mathematical methods and statistics we get to warn people not to gamble.

Some of our objectives are:

1. To find out why casinos can make money through their games.
2. To explain why gambling is so attractive to people using probability theories.
3. To warn people against gamble through educating them on the mathematical approach on their probabilities of winning.

Through the research and study conducted, we can give a preliminary explanation of how casinos make money and what are some traps in gambling games. Meanwhile, we can have sufficient reasons to convince the public not to gamble through our mathematical proof.

2. Introduction:

2.1 Brief introduction

Gambling has always been prevalent all around the world. One of the most significant reasons why such a tremendous number of people are mad about gambling is that they always believe they have a more significant probability of winning money. However, the truth is that the probability of losing money is always much higher than winning. We are interested in such a phenomenon; hence, our project is aiming to find out some interesting facts and extensions about the old saying which originates from China, “If you gamble too much, you are doomed to lose money”. In this way, we hope to warn people not to gamble.

2.2 Research Problems

1. Can people win if they gamble excessively?
2. How to create a game in the same way as a casino?
3. In the game created by Question 2, what are the traps set for players, and what misunderstandings will these traps cause players to go into.

2.3 Objectives

1. To find out why casinos can make money through their games.
2. To explain why gambling is so attractive to people using probability theories.
3. To warn people against gamble through educating them on the mathematical approach on their probabilities of winning

2.4 Rationale

There is a famous proverb in China prevailing among all the gamblers— “If you gamble too much, you are doomed to lose money”. Furthermore, the proverb is not unfounded, the phenomenon that gamblers always leave casinos without much money as they entered. Based on these, our group decided to research in this area, which is pertinent to gambling.

2.5 Field of mathematics

1. Probability Theory
2. Mathematical Statistics

3. Literature review:

3.1 Gambler’s Fallacy

Very brief but accessible first descriptions of the Gambler’s Fallacy itself can be found in many texts, such as Aha! Insight by Gardner, Martin. 1978. Gambler’s fallacy is also known as Monte Carlo Fallacy. In the book, they describe gambler’s fallacy is a mistaken belief about sequences of random events. In other words, the Gambler’s Fallacy is the belief that a “run” or “streak” of a given outcome lowers the probability of observing that outcome on the next trial. In modern society, gambler’s fallacy, however, is usually a method casino taking advantage of people’s psychology to create inclined gambling games.

3.2 Conditional Probability

Kolmogorov, Andrey mentioned conditional probability in Foundations of the Theory of Probability posted in 1956. Conditional probability is a measure of the probability of an event occurring given that another event has occurred. It is usually written as P (A | B). In reality, conditional probability is often ignored by people and thus miscalculating the probability of an event. Casinos make use of the conditional probability and create games that misdirect gamblers so that they cannot tell the probability of winning unless through precise calculation.

3.3 Monty Hall Problem

The Monty Hall Problem gets its name from the TV game show, Let's Make a Deal, hosted by Monty Hall. The Scenario is such: you are allowed to select one closed door of three, behind one of which, there is a prize. The other two doors hide "goats". Once you have made your selection, Monty Hall will one of the remaining doors, revealing that it does not contain the prize. He then asks you if you would like to switch your selection to the other unopened door or stay with your original choice. Here is the problem: does it matter if you switch? This is how we start investigating into conditional probability from the specific example of Monty Hall Problem. In addition, Monty Hall Problem is also a basic model of the games that we have created.

4. Methodology:

1. Theoretically and practically prove that gambling too much can be bound to lose money, using characteristic equations.
2. Explain how casinos use gambler’s fallacy to create gambling games.
3. Explain how casinos use conditional probability to create gambling games.
4. Conclude on the ways casinos creating gambling games and explain why they are unfavourably inclined to gamblers.
5. Using the strategies we discovered, create a game which seems to be fair but is inclined, to make the public better understand these tactics and thus keeping away from gambling consciously.
6. Consider more factors that might affect the judgements of gamblers and try to explore further.
7. Follow the train of thought to solve the extension problem.

5. Working Progress:

5.1 First Stage

After our first discussion, we realised that we could not pick up a real gambling game to do the research. Hence, to solve the first problem, we decided to create an idealised game model to simplify our research. This idealised game is that in this game where a gambler has a winning percentage of x and a bookmaker has a winning percentage of(1-x). The bookmaker will give the gambler one dollar if he wins one game and the gambler will give the bookmaker one dollar if the gambler loses one game. The gambler has A dollar at the very beginning. There are only two termination conditions; one is that the gambler loses all the money, one is that the gambler wins to B dollar, what is the probability of losing all the money.

（Fig5.1）

Idealised game on number axis

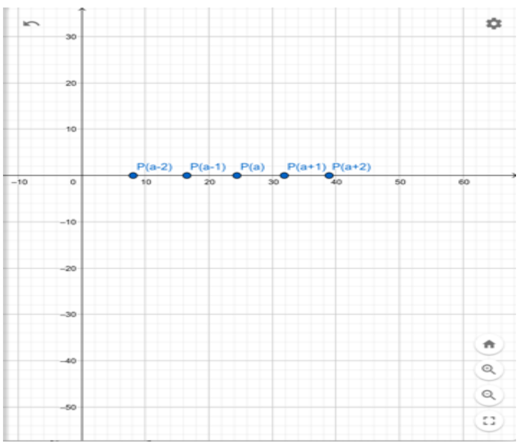


Fig5.1

One of the reasons why so many people are addicted to gambling is that they believe gambling games are fair to both sides--gamblers and casino. Hence, they think they are unlikely to lose money in a fair game. Is that the case? In our idealised game, we assumed that the winning probability of gamblers（varieble X) is 50%. Under this condition, our game is fair for both sides. So, for gamblers, what is the probability of losing all the money?

Our solution to this problem is:

Let the probability of winning to b when the current money is ‘a’ be P(b).

Since winning one game has the same probability as losing one game, which is a. half, after a game, current money the gambler owns is either (a+1) or (a-1).

Therefore, it is not hard to get the equation

$$P(a) = 1/2\ P(a + 1) + 1/2\ P(a - 1)$$

which is equivalent to

$$P(a+1) - P(a) = P(a) - P(a - 1)$$

From this expression, we could clearly tell that the difference of probability to lose all the money between adjacent values of ‘a’ is the same ……*Arithmetic progression*

Furthermore,

P(0) = 1 as the gambler has already lose up his money.

P(B) = 0 as the game has terminated while the gambler is no longer possible to lose all his money.

As the values of P is an arithmetic equation which we could also say as linearly related to the value of 'a'. Therefore, we could conclude on the final expression of P in terms of 'a' —*P(a) = a/B*

In the meantime, gamblers are always too greedy and allured to the gambling games, B is considered infinite for problem gamblers especially. Hence,

$$\lim(b \longrightarrow \infty) P(a) = 0$$

Our solution to fair game told us that in the condition of a fair game, the more a gambler gambles, the higher the probability of losing all his money. As we all know that the winning probabilities of gambling games in casinos can never be 50%, so it is cursory for us to get a general conclusion only through one specific condition. Our next step is to use variable X as an unknown value and do the calculation.

Likewise, to the “fair” game,

$$P(a) = m \times P(a+1) + (1-m) \times P(a-1)$$

which is equivalent to

$$m \times P(a+1) - P(a) + (1-m) \times P(a-1) = 0 \ldots\ldots\ldots\ldots\ldots \textit{(1)}$$

To obtain the general expression of P, we need to resort to a powerful tool called

Characteristic Equation

The characteristic equation of this equation (1) is

$$m \cdot x^2 - x + (1-m) = 0$$

Solving it, we obtain the values of x

$$x_1 = 1 \quad x_2 = \frac{1\text{-}m}{m}$$

Condition (1) ……………… $1 = \frac{1-m}{m}$

Thus, m = 1/2

At this situation, it is a fair game, which is the previous case we have discussed.

Condition (2)

According to the characteristic equations,

$$P(a)=X\cdot 1+Y\cdot\left(\frac{1-m}{m}\right)^{(a-1)}$$

where both X and Y are unknowns.

Likewise, to the "fair" game, substitute the value of a and P(a) by 0, 0 and B, 1 respectively, we get a simultaneous equation.

$$0=X+Y\cdot\left(\frac{m}{1-m}\right) \quad \text{and} \quad 1=X+Y\cdot\left(\frac{1-m}{m}\right)^{(b-1)}$$

Solve the simultaneous equations,

$$X = \frac{m^{b-1}\cdot(1-m)}{(1-m)^b - m^b}$$

$$Y = \frac{m^b}{m^b-(1-m)^b}$$

Thus, substitute the values of X and Y back to the expression of P(a),

$$P(a) = \frac{m^{b-1}\cdot(1-m)}{(1-m)^b - m^b} + \frac{m^{b-a}\cdot(1-m)^a}{(1-m)^b - m^b}$$

$$= \frac{m^{b-a}\cdot\left[m^a-(1-m)^a\right]}{m^b-(1-m)^b}$$

From the expression, it is obvious to see that the value of P(a) is affected by m, a and b simultaneously. To explore the influence of m, a and b on P(a) separately, we shall fix any two of them and observe the influence of the other one factor. At the same time, whether m is greater a half might also influence the results; hence, we still need to discuss the two cases separately.

1.m > 1/2

(1) *"m"* and *"a"* are fixed

We could convert the expression of P(a) to P(a) = $\dfrac{m^{-a}\left[m^{a} - (1 - m)^{a}\right]}{1 - \left(\frac{1 - m}{m}\right)^{b}}$

to help us tell the influence of b on P(a) easily.

As b increases, the value of P(a) decreases.

'b' here represents a termination condition which means the gambler will stop gambling when his money reaches b. Hence, when b is great enough, the gambler is gambling too much, hence, the probability of him losing all his money is getting greater and greater. Therefore, he will be bound to lose money.

(2) *"m"* and *"b"* are fixed

From the original expression of P(a) = $\dfrac{m^{b - a}\left[m^{a} - (1 - m)^{a}\right]}{m^{b} - (1 - m)^{b}}$

We can see that as a increases, P(a) also increases. This means the less original money a gambler owns, the more likely he is going to lose all his money, which is commonplace for everybody.

(3) *"a"* and *"b"* are fixed

From the original expression of P(a) = $\dfrac{m^{b - a}\left[m^{a} - (1 - m)^{a}\right]}{m^{b} - (1 - m)^{b}}$

Likewise, as we can see that as m increases, P(a) also increases. This means the smaller the opportunity to win one gambling game, the more likely a gambler is going to lose all his money, which is also easy to understand.

2. $0 < m < 1/2$

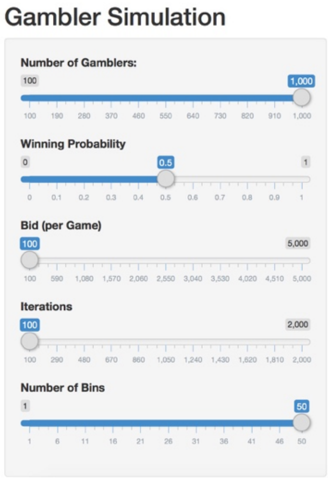
Likewise, through calculation, we obtain the same results as $m > 1/2$.

In conclusion, we finally reach our expectation that people will lose all their money if they gamble too much, which means people are unlikely to win money if they keep gambling. Instead, the result is precisely the opposite.

After having theoretical prove to our conjecture, we decided to make a practice proof for our solution. Following the Large Number theory, we found a gambling simulation to show the justifiability of our solution.

Case 1:

Each gambler has \$10000 at the very beginning. Winning probability of each gambler is 50%. Each gambler bids \$100 per game and plays 100 times. (Fig5.2) The result is Fig5.3. Y-axis represents money, and X-axis represents the number of gamblers. Red line represents the original money each gambler has.



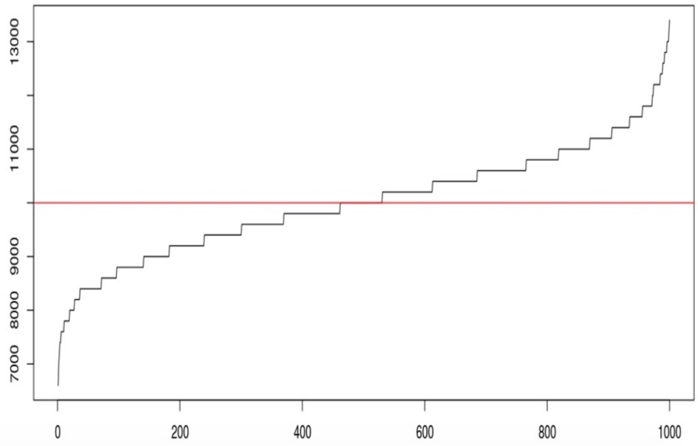


Fig5.2 Fig5.3

The result is that about 500 gamblers lose to less than \$10000

Case 2: (play more and bid more)

Each gambler has $10000 at the very beginning. Winning probability of each gambler is 50%. Each gambler bids $5000 per game and plays 2000 times. (Fig5.4) The result is Fig5.5. Y-axis represents money and X-axis represents the number of gamblers. Red line represents the original money each gambler has.

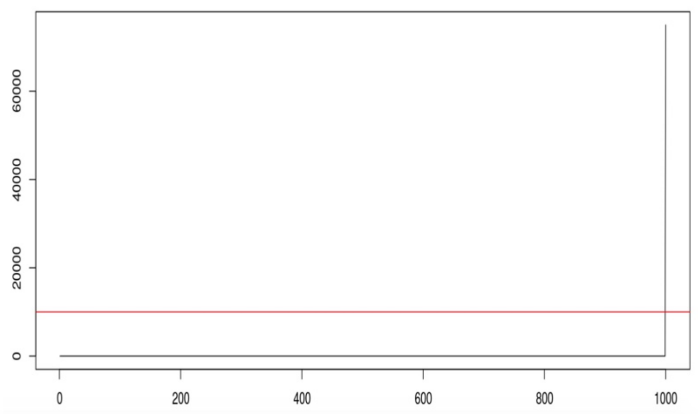


Fig5.4 Fig5.5

The result is that almost every gambler loses to $0, only one or two gamblers can win to more than $10000

Then, if gamblers have a winning probability of 55%, will they win in the end?

Case 3: (winning probability:55%)

Each gambler has $10000 at the very beginning. Winning probability of each gambler is 55%. Each gambler bids $100 per game and plays 100 times. (Fig5.6) The result is Fig5.7. Y-axis represents money and X-axis represents the number of gamblers. Red line represents the original money each gambler has.

Fig5.6

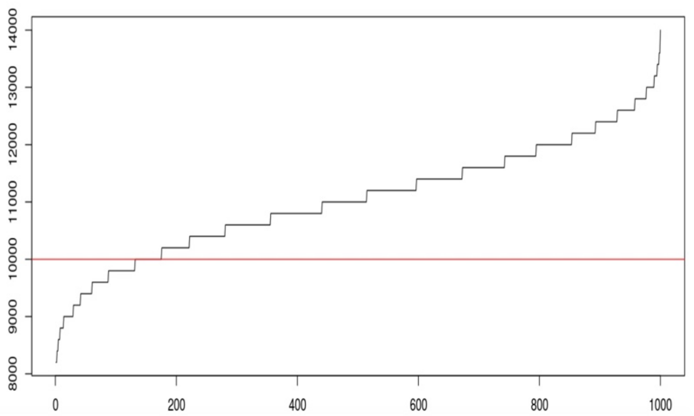


Fig5.7

The result is that if the winning probability for gamblers is 55%, in above condition, more than 800 gamblers will win to more than $10000.

However, if gamblers play more and bid more, it is another story.

Case 4: (winning probability:55% play more and bid more)

Each gambler has $10000 at the very beginning. Winning probability of each gambler is 55%. Each gambler bids $5000 per game and plays 2000 times. (Fig5.8) The result is Fig5.9. Y-axis represents money and X-axis represents the number of gamblers. Red line represents the original money each gambler has.

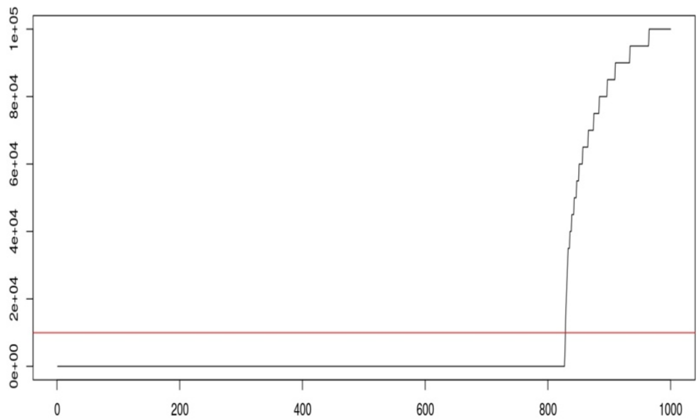


Fig5.8 Fig5.9

The result is that more than 800 gamblers lose to $0.

Our results told us that gamblers would eventually lose out if they make enough bets. We also found that the higher a gambler's odds, the more games it takes to lose all his money. Hence, for casinos, they should reduce the winning probability for gamblers as much as they can, because not every gambler is zealous in gambling.

5.2 Second Stage

After getting our “Law of ‘久赌必输’”,we tried to stand in the casino’s shoes. We attempted to set up some traps to increase our winning probability in a game. Moreover, our game must let gamblers believe that they have a higher probability to win which is opposite to reality.

The first method we found is *the gambler’s fallacy*. In our own words, the gambler's fallacy is the mistaken belief that if something happens more frequently than usual during a given period, it will happen less frequently in the future.

However, when we tried to know further about the gambler’s fallacy, our group had a divergence on one question and the question is like that:

A couple had two children and one of them is a girl. What's the probability that the other child is also a girl ?

Zijian thought：

According to *the gambler’s fallacy*, the possibility of a girl is always 1/2.

There are only two different conditions

A boy or a girl

Zeyu thought：

Assume that two children are child A and child B

There are four different conditions---

(A, B)=(boy, boy)or (boy, girl)or(girl, boy)or(girl, girl)

(boy, boy) is impossible because there is at least one girl, hence the possibility is 1/3

In this case, for Zijian, his understanding of this question is “one specific child is girl”

The whole thing is independent event

P(girl)=1/2

For Zeyu, his understanding of this question is “there is at least one girl”

Hence, the condition of (boy,boy) was rejected at very beginning

P(girl)=1/3

From this, we got our second method to set traps which is *conditional probability*!

In our own words, conditional probability is a measure of the probability of an event occurring given that another event has occurred. Conditional probability is based on *Bayes' theorem*

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(AB)}{\mathcal{P}(B)}$$

After that, we used conditional probability and Bayes' theorem to create a simple model of our game as an example. (Fig5.10)

1.There are four cards on the table.

2.Two are black, two are white

3.You take two in turn

4.If two cards are same, you win

5.If two cards are different, I win.

Fig5.10

Player will think it is a fair game as there are only four possibilities.

Black and black (Fig5.11) *White and white (Fig5.12)*

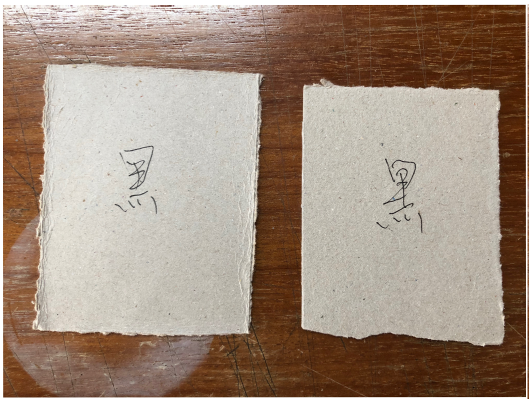


Fig5.11

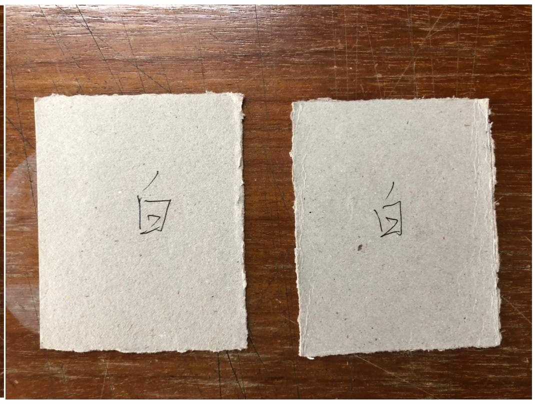
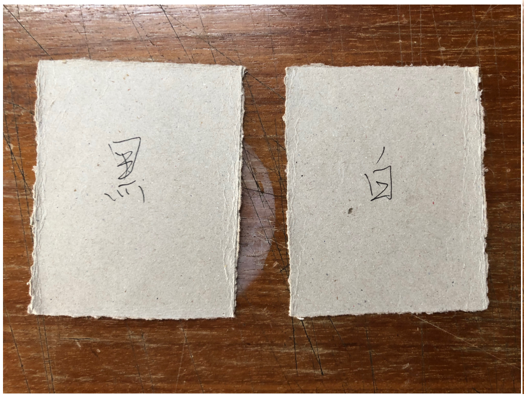


Fig5.12

Black and white (Fig5.13) White and Black(Fig5.14)



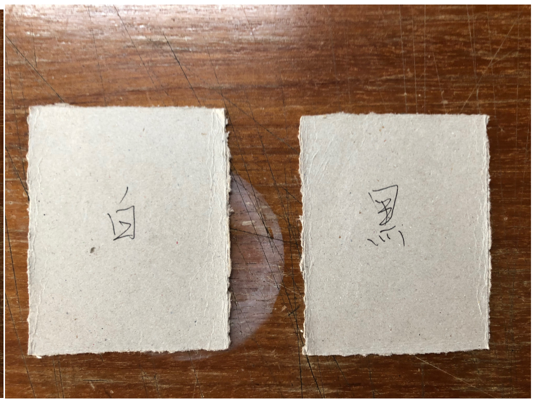


Fig5.13. Fig5.14

Two conditions are same colour, two conditions are different colour. The winning probability looks like 1/2. It seems like a “fair” game.

However, the actual winning probability for player in this game is 1/3.

Inasmuch as once you take away a card, this is what's left (Fig5.15)

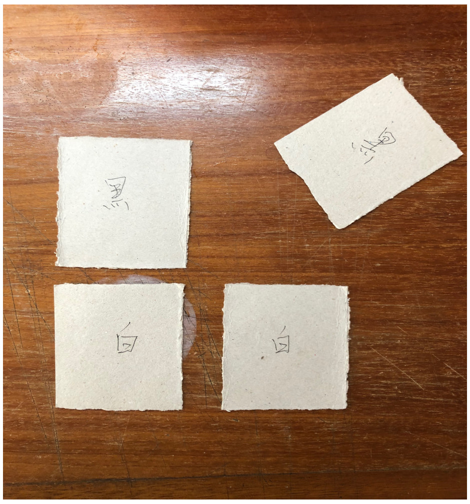


Fig5.15

If you take one black card, there will be one black card and two white cards on the table. Hence, the winning probability for players is always 1/3. And this is a simple model of our game.

Next step is to create a game which is more similar to a real gambling game to show how cunning casinos are, and to warn people not to gamble.

5.3 Third Stage

From our origin “two black and two white” game, we created a more complex game. This game is, in a way, an enhanced version of the previous game.

A handbag contains 8 red balls and 8 black balls (the balls are of the same size). The player pays 1 dollar at a time to take out 8 balls randomly from the handbag, and the number of red balls contained in the balls taken out decides whether to win or lose. For players, the rules are as follows

the number of red balls	0	1	2	3	4	5	6	7	8
money(players can get)	+100	+5	+3	+1	-10	+1	+3	+5	+100

Where "+" means the player wins money, and "-" means the player loses money. For example, if the result is 6 red balls, the player wins 3 dollars. On the face of it, there are nine possible outcomes, eight of which are for the player to win and only one for the player to lose.

If we consider the one dollar that the player paid at the beginning of the game, the actual win and loss of the player is shown in the following table

the number of red balls	0	1	2	3	4	5	6	7	8
money(players can get)	+99	+4	+2	0	-11	0	+2	+4	+99

On the surface, players win money in six conditions, stay in same in two conditions and lose money in only one condition. Players will think that they are bound to win. But this is not the case:

There are 16 balls of the same size in the bag and the player takes any 8 of them.

A_i is to represent that the number of red balls in all 8 balls is i (i=0,1,2……8)

$$P(A_i) = \frac{C_8^{8-i} C_8^i}{C_{16}^8} (i = 1,2,\cdots,8)$$

From this equation, we can get:

$P(A_0) = \frac{1}{12870}$ $P(A_1) = \frac{32}{6435}$ $P(A_2) = \frac{392}{6435}$ $P(A_3) = \frac{1568}{6435}$ $P(A_4) = \frac{490}{1287}$

$P(A_5) = \frac{1568}{6435}$ $P(A_6) = \frac{392}{6435}$ $P(A_7) = \frac{32}{6435}$ $P(A_8) = \frac{1}{12870}$

$P(+99) = P(A_0) + P(A_8) = 0.00016$

$P(+4) = P(A_1) + P(A_7) = 0.00995$

$P(+2) = P(A_2) + P(A_6) = 0.12183$

$P(0) = P(A_3) + P(A_5) = 0.48733$

$P(-11) = P(A_4) = 0.38073$

money(players can get)	+99	+4	+2	+0	-11
probability	0.00016	0.00995	0.12183	0.48733	0.38073

Hence, the winning probability is 0.00016+0.00995+0.12183=***0.13194***

the probability of losing money is ***0.38073***

In this game, the probability of winning is less than the probability of losing money.

Moreover, we can get the mathematical expectation of this game.

$$E(money) = 99 \times 0.00016 + 4 \times 0.00995 + 2 \times 0.12183 - 11 \times 0.38073$$
$$= -3.8887$$

-3.8887 means that if one gambler play too much, he or she will lose \$3.8887 per game.

Hence, we got the final version of our game and we successfully pointed out traps I. this game.

6. Conclusion:

The more a gambler gamble, the more likely he is going to lose all his money. For any gambling game with winning probability of one game being m, if we set up the condition that one would win \$1 if he wins a game and lose \$1 if he loses a game, and he would only stop gambling if he loses all his money or he wins to b dollars, the probability of him losing all his money is P(a) = $\frac{m^{b-a}\left[m^{a}-(1-m)^{a}\right]}{m^{b}-(1-m)^{b}}$. Hence, if he gambles too much, we could even conclude that he is bound to lose money. The reason can be found in the proof.

Gambler's fallacy and conditional probability are two significant tricks used in gambling games to make them seem fair. The gambler's fallacy makes gamblers' think they would definitely win their money back if they lose for the previous rounds. In the meantime, conditional probability gives gamblers a hallucination that the gambling games are fair for both sides and thus attracting them to gamble.

With the aid of gambler's fallacy and conditional probability, we could invent new games, which seem to be fair but are actually not, on our own. Therefore, the public should understand that gambling is risky and dangerous. The best choice for folks is to keep away from gambling thereby.

7.Extension and Future Research:

For future research and extension, firstly, we will try to solve the problem we faced in the process of creating our own game. We will further improve our knowledge about conditional probability and many other gambler's fallacies.

Secondly, we will extend our method to create a more complex game which is the combination of more mathematical traps vis-à-vis the original one.

Thirdly, we try to discover if we can find a general formula to create one specific kind of complex game. Through this formula, we can predict the winning probability of our game before we start to create it.

Finally, we will show more fully how casinos win money from gamblers in the field of mathematics. What we discovered in reality is that some people might allege that they will not gamble that much to lose all their money. In response to this assertion, we want to further warn them not to gamble. Hence, our group intend to extend our field of research to the probability of losing and the amount the gambler will lose on the basis of the previous research on probability of losing all money.

In extension, we could study on different gambling games separately. For each specific gambling game (i.e. a specific probability of losing only one game), and for a specific time that the gambler plays it, we could calculate the probability of losing to a certain amount of money. The amount of money could be flexible with regard to the social economic status of the gambler. Hence, we could use statistics to further persuade people not to gamble as they are even more likely to lose money than losing all the money.

Therefore, we will have sufficient reason to warn people not to gamble

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