PROJECT WORK WRITTEN REPORT

CATEGORY 8

TILES OF MYSTERY

GROUP 8-13

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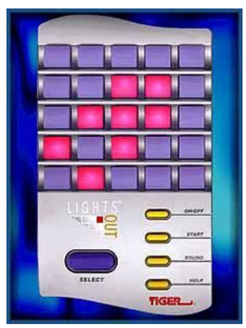
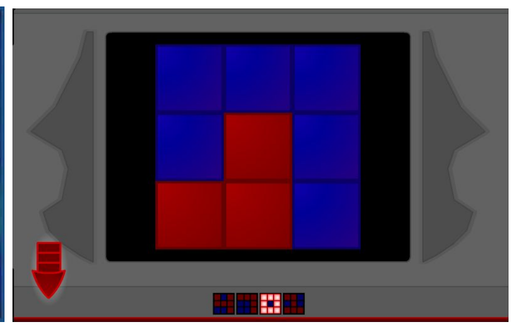
Contents

1 INTRODUCTION

1.1 Background Information

We first encountered this puzzle in a game called Riddle School 5, which is part of the Riddle School series. Upon further research, we found that this type of puzzle was first made by a NASA engineer and his wife in 1978. It was played on a 3 by 3 grid on an electronic device and it was known as “Magic Squares”. Then in 1995, Tiger Electronics made a version of the game called “Lights Out” that was played on a 5 by 5 grid.





Magic Squares Lights Out Riddle School Puzzle

In the Riddle School 5 version, the puzzle consists of 9 squares, which all begin red in colour. Then, when a square is clicked, itself and all horizontally and vertically adjacent squares will switch to the opposite colour, and since they all start as red, they will all switch to blue. The goal of the puzzle is to switch the lights to make specific patterns given by the game.

We found it a challenging puzzle because it was hard to switch one colour without affecting the colours of the tiles around it. This made it difficult to create the pattern given by the game.

We wanted to find out what the logic was behind the game and what patterns the game used when switching colours of only specific tiles, so that there would be a way in which we would always be able to win the game.

Our version of the game is called “Tiles of Mystery”. The goal of the puzzle is to click on a tile to toggle the lights in order to create a set pattern.

1.2 Objectives

The objective of our project is to find the best way to solve the puzzle in the fewest number of moves possible.

1.3 Research Questions

1. Is there an algorithm or formula to create any combination of tiles?

2. Is there only one unique solution for each configuration, or are there multiple solutions?

3. Is there an algorithm or formula to create any combination of tiles for any number of colours?

2 METHODOLOGY

2.1 Methodology

1. We read up material related to how people have tried to solve the “Tiles of Mystery” type of puzzles in the past, and tried to see if we could adapt any of their solutions to help us solve our research questions.

2. We tested out different methods to alter the colour of only one corner, edge or center tile.

3. Based on results obtained from our tests, we observed patterns and attempted to determine the formula for our research questions.

4. We used Algorithms and Algebra to solve the research questions.

5. We experimented with new variables such as increasing the number of colours in the game.

6. We used manual forms as well as C++ coding to check whether our formula would work.

2.3 Literature Review

“All Lights and Lights Out”

Retrieved from:

http://www.iespravia.com/rafa/luces/Lights.pdf

This is quite similar to research question 4. However, the algebraic equation it created to solve a lights out puzzle is dependant on the number of colours so it isn’t general enough to find to create a formula which solves any number of colours.

http://www.logicgamesonline.com/lightsout/tutorial.html

Retrieved 2006

This website is a simple guide on solving the lights out puzzle. However, the guide is limited to 2 colours only and is not very precise and does not always result in the fewest possible moves.

3 RESULTS AND FINDINGS

3.1 Research Question 1 Solution

Let

1	2	3
4	5	6
7	8	9

represent each of the tiles. {1, 3, 7 and 9} are all corner tiles, {2, 4, 6 and 8} are edge tiles, and {5} is the center tile.

CORNERS:
If we want to change only the colour of a corner tile, ie, {1}, we must know the opposite corner, the corner on the opposite side of the grid, in this case, it is {9}. By clicking all the tiles in the same row and column as {9} (except for {9} itself) and also clicking {1}, only the status of {1} will change. (FIGURE 1.1)

EDGE:
If we want to change only the colour of an edge tile, ie, {2}, we must click all the tiles on the other edge, in this case, {7, 8 and 9}, and also click the center tile, {5}. (FIGURE 1.2)

CENTER:
If we want to change only the colour of the center tile, {5}, we must click all the edge pieces, {2, 4, 6 and 8}, and also click {5} itself. (FIGURE 1.3)

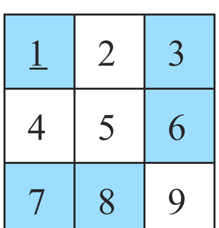


FIGURE 1.1

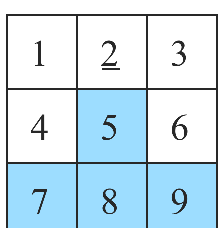


FIGURE 1.2

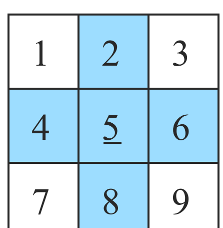


FIGURE 1.3

Underline means tile to be alternated, blue colour represents tiles to be CLICKED to alternate the underlined tile.

To solve a configuration, ie,
, we can count how many
times to click each tile to change each colour.

1	2	3
4	5	6
7	8	9

For example, for {1} we have to click {1, 3, 6, 7 and 8}.
For {3}, we have to click {1, 3, 4, 8 and 9}.
For {4}, we have to click {3, 5, 6 and 9}.
For {5}, we have to click {2, 4, 5, 6 and 8}.
For {6}, we have to click {1, 4, 5 and 7}.
For {9}, we have to click {2, 3, 4, 7 and 9}.

If we count how many times each tile was clicked, we get this:

(3)	(2)	(4)
(4)	(3)	(3)
(3)	(3)	(3)

Clicking a tile an even number of times will have the same result as not clicking it, and clicking a tile an odd number of times will have the same result as clicking it once. So, clicking a tile if it has an odd number and not clicking a tile if it has an even number would get us our solution to the configuration.

The tiles which will be clicked an odd number of times are in blue, and clicking only those tiles WILL get us to the configuration.

3.2 Research Question 2 Solution

Is there only one unique solution for each configuration, or are there multiple solutions?

Let us first consider the case of only two colours.

The number of configurations that can be made is 512, as each of the nine tiles will be either blue or red, and 2 ^ 9 = 512.

The number of possible solutions, or ways to click the tiles, is also 512, as each of the nine tiles will either be clicked or not clicked, and 2 ^ 9 = 512.

As established in question 2, we know that all 512 configurations are possible to solve, and will have at least one solution each. However, there are also 512 possible ways to click the tiles. Thus each of them must solve one of the 512 configurations, and we can safely say that every configuration only has one solution each.

This stays the same for three colours and the rest, as the number of configurations and the number of solutions will both always be X ^ 9, X being the number of colours.

3.3 Research Question 3 Solution

Is there an algorithm or formula to create any combination of tiles for any number of colours?

Every configuration is made out of corners, edges and tiles.

Firstly, let us try to find formulas to change the colour of only one corner, edge or center tile once. These formulas can then be combined to solve for any configuration of tiles.

Let us observe the patterns for changing the colour of only one corner, edge or center once for 2 colours and 3 colours.

2 COLOURS:

Let

1	2	3
4	5	6
7	8	9

represent each of the tiles. {1, 3, 7 and 9} are all corner tiles, {2, 4, 6 and 8} are edge tiles, and {5} is the center tile.

CORNERS:
If we want to change only the colour of a corner tile, ie, {1}, we must know the opposite corner, the corner on the opposite side of the grid, in this case, it is {9}. By clicking all the tiles in the same row and column as {9} (except for {9} itself) and also clicking {1}, only the status of {1} will change. (FIGURE 1.1)

EDGE:
If we want to change only the colour of an edge tile, ie, {2}, we must click all the tiles on the other edge, in this case, {7, 8 and 9}, and also click the center tile, {5}. (FIGURE 1.2)

CENTER:
If we want to change only the colour of the center tile, {5}, we must click all the edge pieces, {2, 4, 6 and 8}, and also click {5} itself. (FIGURE 1.3)

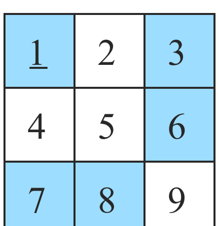


FIGURE 1.1

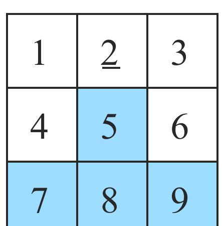


FIGURE 1.2

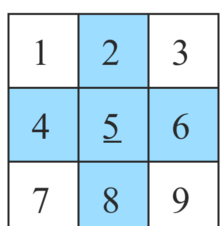


FIGURE 1.3

Underline means tile to be alternated, blue colour represents tiles to be CLICKED to alternate the underlined tile.

3 COLOURS:

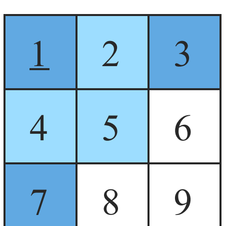
1	2	3
4	5	6
7	8	9

represent each of the tiles. {1, 3, 7 and 9} are all corner tiles, {2, 4, 6 and 8} are edge tiles, and {5} is the center tile.

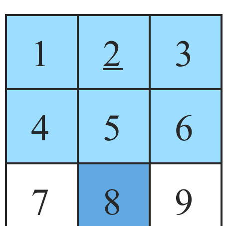
CORNERS:
If we want to change only the colour of a corner tile to the next colour, ie, {1}, we need to click all tiles in the same row and column, including the square itself, ie, {1, 2, 3, 4, 7}. Then, click the center square, ie, {5}, then find the opposite corner, ie, {9}. Then, click all the other corners, ie, {1, 3, 7}. This will leave us with the square {1} changed to the next colour.

EDGE:
If we want to change only the colour of an edge tile, ie, {2}, we have to click all the tiles once, except tiles on the opposite side, which are {7, 8, 9}. Instead, we will click the edge tile directly opposite to our edge tile twice, ie, {8}.

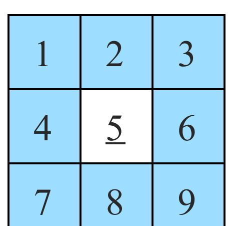
CENTER:
If we want to change only the colour of an edge tile, {5}, we have to click all the tiles once, except {5} itself.



FIGURE



FIGURE



FIGURE

2.1	2.2	2.3

Underline means tile to be alternated, light blue colour represents tiles to be clicked ONCE, and dark blue colour represents tiles to be clicked TWICE to alternate the underlined tile.

Now that we have observed the solutions for any configurations for two colours and three colours, let us now try to find some observations in these solutions to find the solution for any number of colours.

OBSERVATION 1: If the tile to be solved is affected by other tiles equally, those tiles can be classified together and will be clicked and equal number of tiles to change the status of the tile to be solved. For example, when solving for the center tile, the edge tiles are all classified together as they all affect the center tile once. The corner tiles are classified together as they each affect two edge tiles twice.

CENTER:

A	B	A
B	<u>C</u>	B
A	B	A

Assume the tiles labeled the same letter be clicked the same number of times.

A lower-case “a” denotes the number of times each tile labeled “A” is clicked, and an upper-case “A” denotes the number of times the tile labeled “A” changes colour.

X denotes the number of colours in a certain scenario.

Y | Z denotes that Y is divisible by Z.

$A = a + 2b$
$B = 2a + b + c$
$C = 4b + c$

For a tile to not change colour, it must either change colour 0 times, or change colour by a multiple of X number of times, as X is the number of colours and changing colour X times reverts it back to the original colour.

So,
①, $(a + 2b) \mid X$
②, $(2a + b + c) \mid X$
③, $(4b + c - 1) \mid X$

2① - ② + ③
>> $(2a + 4b) - (2a + b + c) + (4b + c - 1) \mid X$
>> ④, $7b - 1 \mid X$

7① - 2④
>> $(7a + 14b) - (14b - 2) \mid X$
>> $7a - 2 \mid X$

7② - 14① - 3④
>> $(14a + 7b + 7c) - (14a + 28b) - (21b - 3) \mid X$
>> $7c + 3 \mid X$

Thus, to find the solution to change the colour of the center piece only once, we only have to find values for a, b and c which satisfy the expressions $7a - 2 \mid X$, $7b - 1 \mid X$ and $7c + 3 \mid X$.

Thus for X being any multiple of 7, it will not be possible to solve all configurations as no such values for a, b, and c can satisfy the expressions.

EDGES:

K	<u>J</u>	K
M	L	M
O	N	O

Assume the tiles labeled the same letter be clicked the same number of times.

A tiny letter “j” denotes the number of times each tile labeled “J” is clicked, and a big letter “J” denotes the number of times the tile labeled

“J” changes colour.

X denotes the number of colours in a certain scenario.

Y | Z denotes that Y is divisible by Z.

$J = j + 2k + l$
$K = j + k + m$
$L = j + l + 2m + n$
$M = k + l + m + o$
$N = l + n + 2o$
$O = m + n + o$

So,
①, $(j + 2k + l - 1) \mid X$
②, $(j + k + m) \mid X$
③, $(j + l + 2m + n) \mid X$
④, $(k + l + m + o) \mid X$
⑤, $(l + n + 2o) \mid X$
⑥, $(m + n + o) \mid X$

-2① + ② + ③ + 3④ -2⑤ + ⑥,
>> $(-2j -4k -2l + 2) + (j + k + m) + (j + l + 2m + n) + (3k + 3l + 3m + 3o) + (-2l - 2n - 4o) + (m + n + o)$
>> $7m + 2 \mid X$

-2① + 8② + ③ - 4④ + 5⑤ - 6⑥,
>> $(-2j -4k -2l + 2) + (8j + 8k + 8m) + (j + l + 2m + n) + (-4k - 4l - 4m - 4o) + (5l + 5n + 10o) + (-6m - 6n - 6o)$
>> $7j + 2 \mid X$

4① - 2② - 2③ + ④ - ⑤ + 5⑥,
>> $(4j + 8k + 4l - 4) + (-2j - 2k - 2m) + (-2j - 2l - 4m - 2n) + (k + l + m + o) + (-l - n - 2o) + (5m + 5n + 5o)$
>> $7k - 4 \mid X$

- 3① + 5② - 2③ + ④ + 4⑤ - 2⑥

$>> (-3j - 6k - 3l + 3) + (5j + 5k + 5m) + (-2j - 2l - 4m - 2n) + (k + l + m + o) + (4l + 4n + 8o) + (-2m - 2n - 2o)$
$>> 7o + 3 \mid X$

5① - 6② + ③ - 4④ - 2⑤ + 8⑥,
$>> (5j + 10k + 5l - 5) + (-6j - 6k - 6m) + (j + l + 2m + n) + (-4k - 4l - 4m - 4o) + (- 2l - 2n - 4o) + (8m + 8n + 8o)$
$>> 7n - 5 \mid X$

① - 4② + 3③ + 2④ + ⑤ - 4⑥
$>> (j + 2k + l - 1) + (- 4j - 4k - 4m) + (3j + 3l + 6m + 3n) + (2k + 2l + 2m + 2o) + (l + n + 2o) + (-4m - 4n - 4o)$
$>> 7l - 1 \mid X$

Thus, to find the solution to change the colour of the edge piece only once, we only have to find values for j, k, l, m, n and o which satisfy the expressions $7j + 2 \mid X$, $7k - 4 \mid X$, $7l - 1 \mid X$, $7m + 2 \mid X$, $7n - 5 \mid X$ and $7o + 3 \mid X$.

Thus for X being any multiple of 7, it will not be possible to solve all configurations as no such values for j, k, l, m, n and o can satisfy the expressions.

CORNERS:

<u>D</u>	E	G
E	F	H
G	H	I

Assume the tiles labeled the same letter be clicked the same number of times.

A tiny letter "d" denotes the number of times each tile labeled "D" is clicked, and a big letter "D" denotes the number of times the tile labeled "D" changes colour.

X denotes the number of colours in a certain scenario.

Y | Z denotes that Y is divisible by Z.

$D = d + 2e$
$E = d + e + f + g$

F = 2e + f + 2h
G = e + g + h
H = f + g + h + i
I = 2h + i

So,
①, (d + 2e - 1) | X
②, (d + e + f + g) | X
③, (2e + f + 2h) | X
④, (e + g + h) | X
⑤, (f + g + h + i) | X
⑥, (2h + i) | X

4① - 4② + ③ + ④ + 3⑤ - 3⑥,
>> (4d + 8e - 4) + (- 4d - 4e - 4f - 4g) + (2e + f + 2h) + (e + g + h) + (3f + 3g + 3h + 3i)+ (- 6h - 3i)
>> 7e - 4 | X

- ① + 8② - 2③ - 2④ - 6⑤ + 6⑥,
>> (-d - 2e + 1) + (8d + 8e + 8f + 8g) + (-4e - 2f - 4h) + (-2e - 2g - 2h) + (-6f - 6g - 6h - 6i)+ (12h + 6i)
>> 7d + 1 | X

- ① + ② - 2③ + 5④ + ⑤ - ⑥,
>> (-d - 2e + 1) + (d + e + f + g) + (-4e - 2f - 4h) + (5e + 5g + 5h) + (f + g + h + i)+ (-2h - i)
>> 7g + 1 | X

- 2① + 2② + 3③ - 4④ + 2⑤ - 2⑥,
>> (-2d -4e + 2) + (2d + 2e + 2f + 2g) + (6e + 3f + 6h) + (-4e - 4g - 4h) + (2f + 2g + 2h + 2i)+ (-4h - 2i)
>> 7f + 2 | X

-3① + 3② + ③ + ④ - 4⑤ + 4⑥,
>> (-3d - 6e + 3) + (3d + 3e + 3f + 3g) + (2e + f + 2h) + (e + g + h) + (-4f - 4g - 4h - 4i)+ (8h + 4i)
>> 7h + 3 | X

6① - 6② - 2③ - 2④ + 8⑤ - ⑥,

>> $(6d + 12e - 6) + (-6d - 6e - 6f - 6g) + (-4e - 2f - 4h) + (-2e - 2g - 2h) + (8f + 8g + 8h + 8i) + (-2h - i)$
>> $7i - 6 \mid X$

Thus, to find the solution to change the colour of the edge piece only once, we only have to find values for d, e, f, g, h and i which satisfies the expressions $7d + 1 \mid X$, $7e - 4 \mid X$, $7f + 2 \mid X$, $7g + 1 \mid X$, $7h + 3 \mid X$ and $7i - 6 \mid X$.

Thus for X being any multiple of 7, it will not be possible to solve all configurations as no such values for d, e, f, g, h and i can satisfy the expressions.

Thus, to solve any configuration for any number of colours, we need to solve how to change the colour of any corner, edge or tile once, by finding values for a to o which satisfies the expressions, which will be different depending on the number of colours. A copy of the C++ program and the manual drafts have been attached in the appendix.

4 CONCLUSION

4.1 Limitations

We have found that it is possible to solve for any configuration for any number of colours, except for multiples of 7. This is because for any variable, such as a, the expression needed to solve for a is $7a + y \mid X$ (y varies based on the variable used), and if y is not a multiple of 7, $7a + y$ cannot be divisible by a multiple of 7.

Also, if we increase the scale to 4 x 4 and 5 x 5, not all configurations are possible to solve.

4.2 Extensions

A possible extension to this research project would be to increase the scale of our project. An example below is the solution for increasing the grid to 6 x 6, for 2 colours. Other possible extensions would be to change the shape of the grid from squares to other shapes, like triangles and hexagons, or possibly making it 3 x 3 x 3, a 3 - dimensional puzzle.

1		1		1	1
		1			
1	1		1	1	1
		1		1	
1		1	1		
1		1			

				1	1
	1		1		
1	1		1		1
	1				1
		1	1	1	
			1		

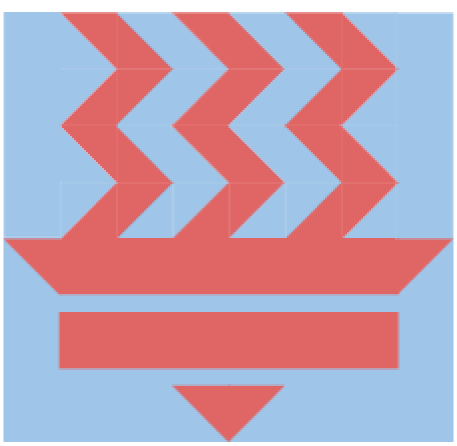
1		1	1		
1		1		1	
			1	1	1
1					
1	1		1	1	1
1				1	

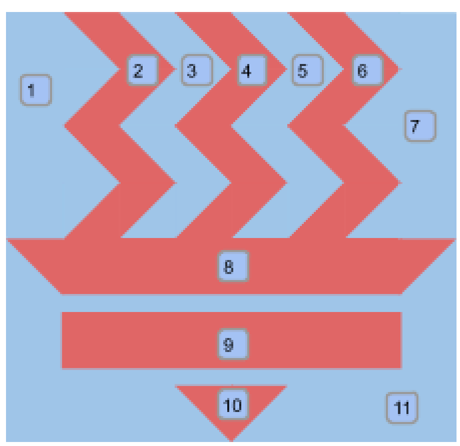
	1		1		
1	1		1	1	
			1		1
1	1	1		1	1
	1		1		1
		1	1	1	

1		1		1	
1		1		1	1
		1		1	
1	1				
				1	
1	1		1	1	1

			1	1	1
		1		1	
	1	1	1		
1		1			
1	1				
1					

Next, we can also try to solve a puzzle in the shape of a hwa chong logo. An example below is the solution for 2 colours.





Each number on the left represents one of the tiles in the above picture, and the way to solve each of those tiles is by clicking the tiles labeled with the numbers on the right.

1	3 6 8 9 10 11
2	3 4 6 7
3	1 2
4	2 6 8 9 10 11
5	6 7
6	1 2 4 5
7	2 5 8 9 10 11
8	1 4 7 8 9 10 11
9	1 4 7 8 9
10	1 4 7 8 9 10
11	1 4 7 8

4.3 Acknowledgements

We would like to thank our project mentor, Mrs Chua, who has guided us through the course of this entire project. She has helped point us in the right direction on several occasions so we know what we need to keep doing. We also want to thank the judges in the mid-term evaluations, for their insightful and valuable feedback to help us improve. In addition, we would like to thank Hwa Chong Institution for giving us this opportunity to work on this research project.

4.4 References

http://www.iespravia.com/rafa/luces/Lights.pdf

http://www.logicgamesonline.com/lightsout/tutorial.html

4.4 Appendix

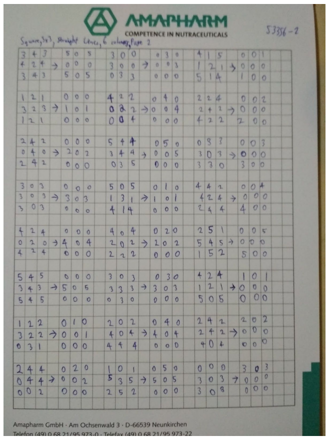
We have included our C++ code and our manual calculations.

C++ code:

```
int find (int z){

        if (z != 0) z = abs(z) % x;

        for (i = 0; i <= x; i ++){
                if (7 * i % x == z){

                        i = i % x;
                        break;

                }
        }
}

int main(){

        int a, b, c, d, e, f, g, h, i, j, k, l, m, n, o;

        cout << "No. of colours: ";

        while (cin >> x && x != 0){

                a = find (x - 2);
                b = find (1);
                c = find (3);
```

```
            d = find (x - 1);
            e = find (4);
            f = find (x - 2);
            g = find (x - 1);
            h = find (x - 3);
            i = find (6);

            j = find (x - 2);
            k = find (4);
            l = find (1);
            m = find (x - 2);
            n = find (5);
            o = find (x - 3);

            cout << "CORNER:" << endl << d << " " << e << " " << g << endl
<< e << " " << f << " " << h << endl << g << " " << h << " " << i << endl <<
endl << "EDGE:" << endl << k << " " << j << " " << k << endl << m << " " << l
<< " " << m << endl << o << " " << n << " " << o << endl << endl <<
"CENTER:" << endl << a << " " << b << " " << a << endl << b << " " << c << "
" << b << endl << a << " " << b << " " << a << endl << endl << endl << "No. of
colours: ";
      }
}
```

Manual Calculations: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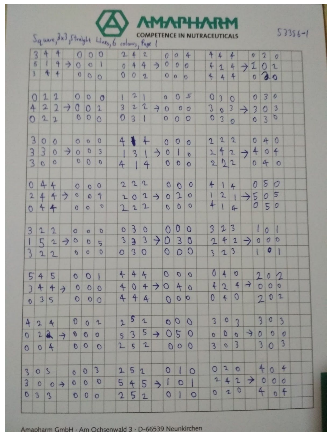
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