

# Blokus Winnus

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# 1. Introduction

## 1.1 Aim

Blokus is a game created in 2000 by Sekkoïa, a French company. It has since won many awards, including the Mensa Select Award and the 2004 Teacher's Choice Award. A game that is highly based on strategy and planning ahead, our group wanted to investigate the game and find out the best strategy to beat the game, and in the process help players learn abilities such as farsightedness. We also hope this project will give more meaning to the game such that players do not just place blocks without thinking but instead think more critically.

## 1.2 Gameplay

### 1.2.1 Tiles and Pentominoes

A tile is a small square on the square grid of the board, while pentominoes are pieces a player can place to make a move.

For example, in Fig. 1, you can see one red pentomino on a grid of 12 white tiles.

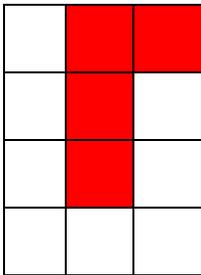


Fig 1. A pentomino on a grid of 12 tiles.

There are a total of 21 pentominoes a player has, and we have subsequently given each of them a name where we will be using throughout the report. We have named the tiles such that the name will correspond, somewhat, to the shape and size of the pentomino.

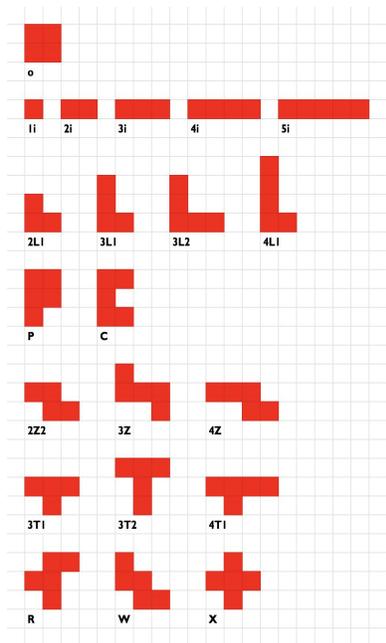
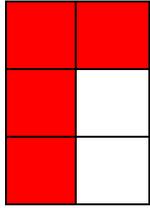


Fig 2. a diagram of all the pentominoes with their corresponding names.

As you can see, for example, the 5-by-1 tile is grouped under the “i” category simply because it looks like a 5-tile-long I. Therefore its name would be 5i.

Another example would be the 4L1 tile. The 4L1 is grouped under the “L” category due to its L-shaped configuration and we since it has 4 blocks tall and it protrudes out 1 block from the side, its name is 4L1.

The tile area of a pentomino is the number of tiles a pentomino takes up when placed. The inverted 3L1 tile below, for example, takes up a tile area of 4.



### 1.2.2 Starting the game

An official Blokus board has 400 tiles, and each player will start with 21 pentominoes. Each pentomino takes up 1 to 5 tiles of space, and the total space taken up by all of one player's pentomino is 89 tiles. A player starts his first move by playing a pentomino on one of the top corners of the board.

### 1.2.3 Making a move

A player makes a valid move by placing another pentomino diagonally to another pentomino the player themselves has already placed down. Additionally, the pentomino must be placed in an area in which no pentominoes from any player have been placed before. A move is considered invalid when it does not touch the player's existing pentominoes, or if it touches the player's own tile vertically or horizontally. To clarify, a player's pentomino can touch another pentomino in any orientation, if and only if it is not the player's own pentomino.

Referring to figure 3a and 3b, if red were to place a 2L1 pentomino in the area highlighted in light red in Figure 3a, it is considered legal even if it is touching the blue tile vertically and horizontally as it only touches its own pentomino diagonally in an area that no pentomino had been placed before.

However, if the player in red were to place the 3L1 pentomino in Figure 3b, it would be an invalid move because it is touching its own tile vertically.

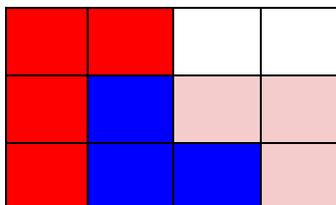


Fig. 3a, a valid move

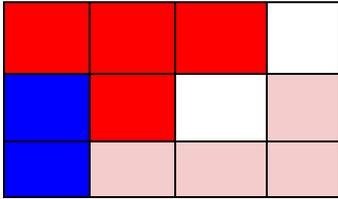


Fig. 3b, an invalid move

### 1.2.4 Ending the game

The game ends when no more player can place a pentomino on the board. The winner of the game is the one who has the least number of tile area in a pentomino. Figure 4 shows an example of an end-game state.



Fig. 4, an end-state.

Here, the purple player is the winner, as they have a combined area of 11 left.

## 1.3 Literature Review

### 1.3.1 Literature Review 1

Hart, E. (n.d.). Blokus Strategy. Retrieved July 30, 2019, from <http://blokusstrategy.com/>

A blog written by a former top player of Blokus.

It is a very in depth guide of Blokus that goes through a lot of things, from basics like piece names and how to open the game, to advanced strategies like knowing which fights to take and avoid, and which pieces are the most useful

However, it is quite limited as it only goes through strategies used in 1v1 battles, it does not really touch on 1v1v1v1 battles, or 2v2 battles.

### 1.3.2 Literature Review 2

Kershner, K. (2012, March 21). How Blokus Works. Retrieved July 30, 2019, from <https://entertainment.howstuffworks.com/leisure/brain-games/blokus2.htm>

This is a simple guide to the basics of Blokus, it only states the basic tactics of blokus, and does not go much into them, the writer talks mostly about why blokus is a good game to play, and how it captures his attention.

## 2. Research Questions

Our research questions are as follows:

1. What is the smallest square grid in which all pieces of tile area 4 and below can be placed?
2. What are the most important tiles to play at certain times of the game if it is a 1v1v1v1 game?
3. What, if any, is a good strategy in a 2v2 game?

Research questions 1, 2 and 3 will be answered in sections 3, 4 and 5 respectively

## 3. What is the smallest square grid in which all pieces of tile area 4 and below can be placed?

### 3.1 Lower bound

Since we are only employing pentominoes with only tile area 4 and below, we are removing all pentomino with tile area 5.

Thus, we only remain with 9 pentominoes a player can work with, namely: O, 1i, 2i, 3i, 4i, 2L1, 3L1, 2Z2, 3T1 (Refer to Figure 2). In total they have a tile area of 29.

Since there are 4 players on the board, the board has to be at least

$$29 \times 4 = 116$$

116 tiles large.

Since we are working with a square grid, the lowest square number that is larger than 116 is  $11^2$ , or 121. Thus, this is the lower bound.

### 3.2 Upper bound

#### 3.2.1 Tile consumption

Before we solve the upper bound we will first explain the meaning of tile consumption.

When a player places down a piece, they will inadvertently block out certain tiles for them to make other moves when we place a pentomino.

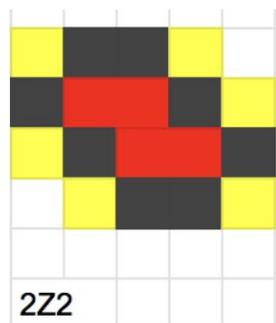


Fig. 5, an example of the tiles blocked out for future use.

In Figure 5, the red area is the pentomino 2Z2, while the grey area shows the area where a player cannot make a move anymore

without making an invalid move. The yellow area shows the area where they can make a next move.

Additionally, when a tile is placed they will, obviously, block out the area it is sitting on. Thus, tile consumption is the addition of the red area and the grey area.

We then quantified the tile consumption of all the pentominoes that were relevant to the question.

Pentomino	Tile Consumption
O	12
1i	5
2i	8
3i	11
4i	14
2L1	10
3L1	16
2Z2	12
3T1	12

Finding the total sum of all of the pentominoes available to all the players in the premise of the research question, we have the total number of tiles consumed to be:

$$12 + 5 + 8 + 11 + 14 + 10 + 16 + 12 + 12 = 100$$

thus, for all 4 players, they have a total tile consumption of 400.

### 3.2.2 Solution

Assuming all the players do not play into others' tile consumed areas, in other words making it a worse-case scenario as players are all playing as if they are the same player, a 400 tile board is needed.

As 400 is exactly  $20^2$ , we need a 20-by-20 grid to suit this configuration.

### 3.3 Conclusion

Thus, using the lower bound and upper bound, we first tried fitting all of such tiles with an 11-by-11 grid, which worked.

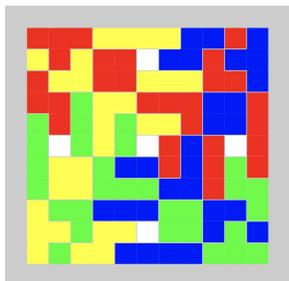


Fig. 6, an 11-by-11 tile grid configuration.

As shown in Figure 6, an 11-by-11 grid is sufficient to fit all of the 9 pentominoes per player that fit the criteria of a tile area less than 4.

### 3.4 Possible extensions

As tiles with tile area 5 take up the majority of tile area among all the pentominoes, we suggest extensions can be made on the smallest square grid that can fit all the pentominoes of tile area 5. Additionally, another extension can be to find a board of 116 tiles that can fit all the tiles without any extra leftover, of if that is possible.

## 4. What are the most important pentominoes to play at certain times of the game if it is a 1v1v1v1 game?

A game of Blokus has 3 phases: although not very clear cut, are relatively always forced and present due to the availability of tiles. The 3 stages are as follows: expansion, sealing off, and conservation. A player will most likely expand in the first stage due to the huge amount of space available at the start and the need to save blocks in the later game. A player will have to seal off at some point due to all players expanding and into each other's territories, and conservation will be needed at the end due to the lack of available tiles at the end.

Thus, we have split the game, and the most important tiles of each phase, into three parts as stated above: Expansion, Sealing off and Conservation. There is a different counting and ranking method for each of the three stages, and we will be expanding further on these.

The value of all the pentominoes in the respective measurements can be found at **Appendix A**.



## 4.1 Expansion

### 4.1.1 Rubik distance

An idea of a rubik distance originates from a Blokus strategist Rubik87. A rubik distance is, essentially, the length of a tile.

If one thing is  $x$  number of rubik distances away from something else, given that it is  $y$  rubik distances away vertically and  $z$  rubik distances horizontally,  $x = y + z$ .

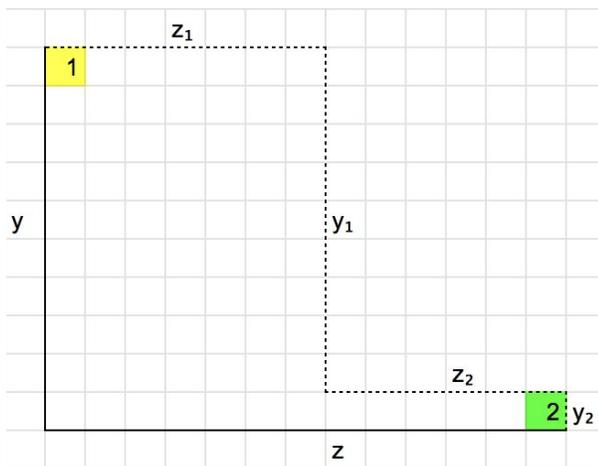


Fig. 7

As shown in Figure 7, if the dotted line shows the actual movement of the pentominoes in the actual game (or  $x$ ), we can always extract line segments  $y_1, y_2, \dots, y_n$  and fit them into line  $y$  because they are parallel and travel equal distances. Similarly, we can extract line segments  $z_1, z_2, \dots, z_n$  and fit them into line  $z$ .

Thus, the statement above holds true.

A pentomino's rubik distance is the number of rubik distances (RD) it takes to travel the shortest route from one corner of the tile to the other.

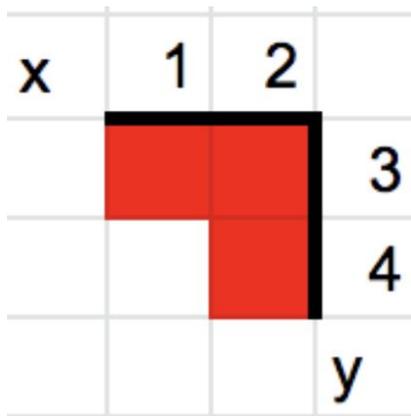


Fig. 8, the RD of pentomino 2L1

As seen here, to travel from point  $x$  to point  $y$ , the two corners of the pentomino, a total of 4 rubik distances is covered.

We will be using rubik distance to rank the different pentominoes to find out the most important pentominoes at this period of time.

### 4.1.2 Way to expand

The most efficient way to expand is to expand diagonally. It expands quicker and gives us more area to work with in the future.

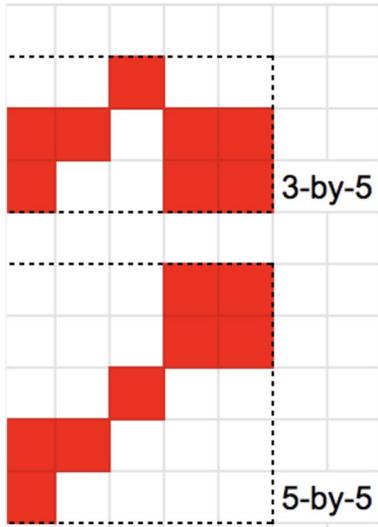


Fig. 9, different ways of expansion.

As shown in Figure 8, when one expands diagonally, we are 5 RD vertically and horizontally away from the starting point, in other words, we are

$$5 + 5 = 10$$

10 RD away from the starting point.

However, if we were to expand in a haphazard fashion, we would only reach

$$3 + 5 = 8$$

8 RD away from the starting point.

Therefore, expanding diagonally is the best way to expand.

### 4.1.3 Most important tiles

We found the rubik distances of each pentomino and here are as follows:

Pentominoes	RD
5i, 3L2, 4L1, 3Z, 4Z, W	6
4i, 3L1, 3T2, 4T1, 2Z2, P, C, R	5
3i, 2L1, 3T1, O, X	4
2i	3
1i	2

Thus, we have found out that the most effective and efficient blocks are the ones with the RD of 6.

Next, the question is how many of such pentominoes should we place before we start the next phase?

The centre of the board is halfway vertically and horizontally across the board, and since the board is 20 tiles long and wide, each player is

$$\frac{20}{2} \times 2 = 20$$

20 tiles away from the centre of the board from their starting point.

How will the average player play?

Finding the average of the RD of all the pentominoes, we have

$$\frac{(6 \times 6) + (5 \times 8) + (4 \times 5) + 3 + 2}{21} = \frac{101}{21} \approx 4.81$$

Assuming the average player plays a little better than the average RD, we will round up 4.81 to 5. Since the centre is 20 RD away from the starting point of the board, the average player is 4 moves away from reaching the centre. Since we have an advantage over the average player in terms of using higher RD blocks, we will take this opportunity to expand more area as compared to them. Because they take 4 turns to reach the centre, we will also play pieces of 6 RD for 4 turns so we have a greater area coverage. As a result, we will be about 24 rubik distances away from the starting point.

What 4 tiles do we use then? As will be touched on later, the tiles 5i and 4L1 are important for future use, thus we will use the 4 remaining tiles: 3L2, 3Z, 4Z, W.

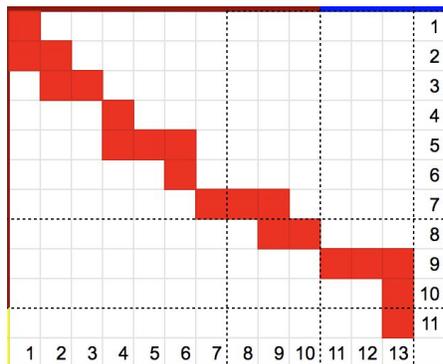


Fig. 10, using all the tiles above.

Figure 10 shows an arrangement where the tiles W, 3Z, 4Z and 3L2 are used (in that order). As shown, they reach a total of 13 RD horizontally and 11 RD vertically, resulting in 24 RD.

Therefore, the tiles 3L2, 3Z, 4Z and W are the most important in this stage.

## 4.2 Sealing off

### 4.2.1 Single and double blocks

A block is defined as a potential block by a pentomino.

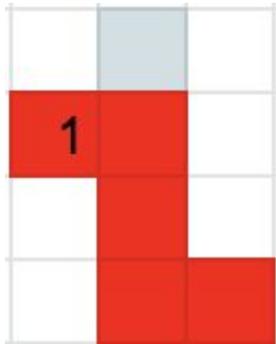


Fig. 11a, a single block.

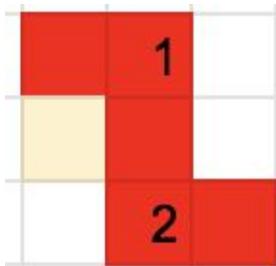


Fig. 11b, a double block.

For example, in Figure 11a, the green tile is potentially blocked in 1 corner, making it a single blocked, while in Figure 11b, the yellow tile is potentially blocked in 2 corners, making it a double block.

We ranked the blocks by double blocks first before single blocks, because double blocks contain significantly more chance of blockage compared to single blocks. Single blocks still open 3 corners for the opponent pentominoes to expand while double only leave 2. Thus, we prioritise double blocks more than single blocks.

Here is our ranking from best to worst, excluding tiles we have already used for expansion:

Tiles	Double	Single
5i	6	8
4i	4	8
4L1	3	11
3T2, 4T1	3	10

C	2	10
3i	2	8
P, R	1	12
3L1	1	11
3T1	1	10
O, 2Z2, X	0	12
2L1	0	10
2i	0	8
1i	0	4

#### 4.2.2 Most important blocks

After expansion, we will be 24 RD away from the centre. Since the RD for a 1-by-1 grid is 2, after we have to expand diagonally outwards after expansion before we can seal off, we will be at least 26RD out the starting space.

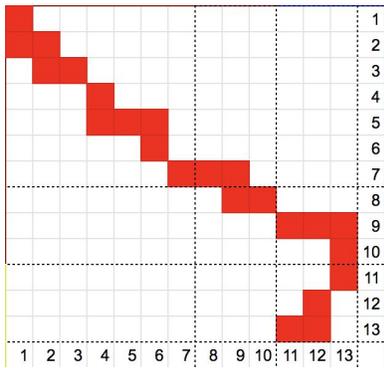


Fig. 12, an example of such a move.

In Figure 12, after we have placed all the tiles for expansion plus one more dummy move, we have to defend our area so we can proceed to place down our remaining blocks without opposing players infiltrating our territory. Thus, given that we have expanded 26 RD, we have to expand along the perimeter of our “territory”, marked as the outermost dotted lines.

As we have shown in 4.1.1., since we are 26 RD away from the starting point, the perimeter of our area is 26 (excluding those at the top and left sides, as they are the board boundaries), since  $26 = y + z$ , where  $y$  is the vertical distance, or the dotted line on the right side in Figure 12, while  $z$  is the horizontal distance, which is the dotted line on the bottom side.

As such, the player needs to flip the pentominoes on their long side so to cover the required distance quickly. After taking the length of the longest sides of the top rankers in blockage (refer to the table in 4.2.1), we found that we need 7 pentominoes to successfully defend the distance of 26RD: 5i, 4i, 4L1, 3T2, 4T1, C, 3i.

Thus, these are the most important pentominoes of this stage.

### 4.3 Conservation

Conservation is measured in terms of tile consumption (mentioned and explained in **3.2.1**). Each pentominoes has a different number of tiles consumed. Additionally and fortunately, when compared to the rankings of rubik distance and blockage, conservation has almost an inverse-correlation with them save a few pentominoes. Thus, it is overall safe to use the remaining blocks when conserving.

Remaining Tiles	Tile Consumption	Ranking for Tile Consumption
1i	5	1 <sup>st</sup>
2i	8	2 <sup>nd</sup>
2L1	10	3 <sup>rd</sup>
3T1	12	5 <sup>th</sup>
2Z2	12	5 <sup>th</sup>
O	12	5 <sup>th</sup>
X	13	9 <sup>th</sup>
P	14	10 <sup>th</sup>
R	16	18 <sup>th</sup>
3L1	16	18 <sup>th</sup>

As we can conclude from the results, the pentominoes R and 3L1 do badly in this area. Thus, we suggest the two pentominoes be used first as tiles will get scarcer as the game progresses, resulting in a need of lower tile consumption. In fact, as a result of this, the pentominoes should be used from bottom of the table to the top.

Thus, for the conservation phase, the most important tile is the bottommost tile that is still unused in the above table.

#### 4.4 Simulation and results

We have simulate 25 games each with and without our strategy with AI on <https://blokee.mattle.online/>. Here are our results.

	Tile area left (Player average/3 AI average [Difference])	Player Ranking (1 <sup>st</sup> /2 <sup>nd</sup> /3 <sup>rd</sup> /4 <sup>th</sup> )	Win percentage
With Strategy	7.88 / 20.92 [13.04]	20 / 5 / 0 / 0	80%
Without Strategy	11.88 / 19.10 [7.22]	13 / 6 / 2 / 4	52%

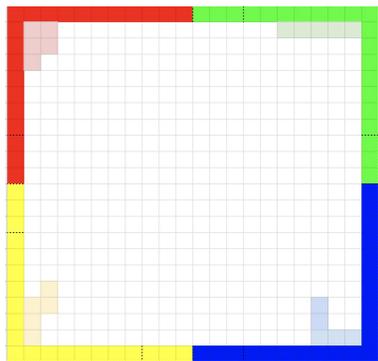
From the results obtained above, through tile area left, we can see that our strategy will beat the 3 AI with a larger difference (5.82 tiles) and forces the AI to lose with more tile area left (1.92 more). With our strategy, we have never dropped below 2<sup>nd</sup> place while without the strategy, we have come in last 4 times. Finally, we win 53% more often with the strategy compared to without.

Thus, our strategy and the most important tile in each phase is functional and able to work.

## 5. What, if any, is a good strategy for a 2v2 game?

### 5.1 Variations compared to standard Blokus

There are certain more freedoms in a 2v2 Blokus game as compared to standard free-for-all Blokus. Such freedoms vary from person to person, but we have decided to set the rules to the most commonplace variation of 2v2 Blokus.



The main difference in this variation is that players of the same team can treat each other tiles as from another side (i.e. their tiles can meet vertically or horizontally). In addition, the two players of the same team play from opposite sides of the board.

Fig. 13, a standard Blokus board.

Using Figure 13 to illustrate, the red and green players are on one team, while the blue and yellow are of another. This is to ensure both sides of the team travel the longest distance possible to meet each other and to force competition in the centre.

Lastly, the winning team is the team with the least amount of *total* tile area left. For example, if the red player has 5 and blue player has 20, while the green player has 7 and yellow player has 19, even though blue would lose in a 1v1v1v1 match, the red-blue team still wins because it has one less tile area (25 compared to 26).

### 5.2 Ideation

There are no articles online for a 2v2 Blokus game, and thus we had to find a new strategy for such a game configuration.

Using the 1v1v1v1 strategy that has been proven successful in **Research Question 2**, we have tried to formulate another strategy to fit a 2v2 Blokus game.

We also realised that since two players can place their tiles side-by-side, we can exploit this by placing as much tiles as possible in this configuration as we can reduce space wastage, as demonstrated in Figure 14.

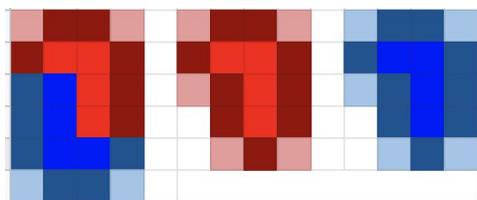
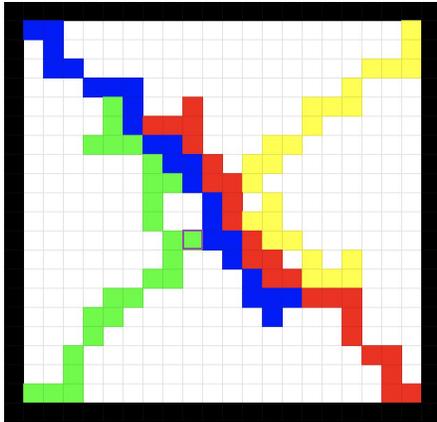


Figure 14, an illustration of blocks placed together compared to blocks placed apart.

In Figure 14, when the two 2L1 pentominoes are placed together to form a 2-by-4 rectangle, only 12 tiles are blocked out (represented by the darker red and blue). However, if the 2L1 were to be placed separately, 9 tiles will be blocked out by each player, giving 18 tiles in total.

Moreover, if we reached the centre quickly and blocked the centre using the following method, we would be able to block out the two opponents from meeting each other, as shown in Figure 15.

Fig. 15, an example of the abovementioned strategy.



In Figure 15, red and blue meet at the centre, and begin to layer their pieces over one another, with each placing their pieces right beside their teammates pieces. This seals off the gaps that are usually present between pieces as pieces of the same colour can only touch corners of their own pieces.

By doing this layering effect, neither of the two opponents can meet, as it removes the possible escapes from a tile in a 1v1v1v1 mode. When we refer back to

**Figure 11b**, even the best “double block” still leaves chances for the opponent to slip through the gaps. As seen in the abovementioned figure, we see an opportunity for the yellow player to place a pentomino on the top left tile (that is off-image).

However, in 2v2 mode, it is essential that teammates work together to produce the best results, with the opponents being unable to team up, your team can then capitalise and defeat either of the players individually, as it would then be two 2v1 situations, instead of a 2v2 situation.

Using this two pieces of knowledge, we will proceed to construct the strategy in section **5.3**.

## 5.3 Strategy

### 5.3.1 Expansion

Since in the two player team the two players have to ideally layer each others’ tiles, when we are expanding toward the centre, the two players cannot use the same tile in the same move, because each player only has one of such tile and cannot layer a tile if one of it has been placed down before.

When we are layering, at the end of a layer, since the same tile cannot be used twice, the other player of the same team will have to end their layer one or two tiles horizontally or vertically away from the start of the other player’s layer. Since the other player’s starting point is 20 tiles vertically and horizontally away, using the equation

$$y + z - 1 = x$$

where the -1 refers to the one tile off in either direction, by substituting  $y$  and  $z$  into the equation (see **4.1.1**), we have:

$$20 + 20 - 1 = 39$$

Thus, each layer will be 39 rubik distances long.

In section 4.1.3, since we have established that there are 6 pentominoes with rubik distance 6, if both sides were to layer all their 6 RD pentominoes, they would be

$$6 \times 6 = 36$$

36 rubik distances away from their starting position. Thus another pentomino with rubik distance 3 can be used. Since there is only 1 tile with rubik distance 3 (i.e. 2i), the tile will be end of the layer.

### 5.3.2 Pentomino 2i

However, using the 2i pentomino, we will encounter some difficulties fitting it in with the final piece.

If we used a checkerboard fashion to colour the board, we will find that pentominoes with an even number of rubik distance would meet the other pentomino at a tile with the same colour while pentominoes with an even number of rubik distance will meet at the opposite colour. This is because the number of tiles to cover the rubik distance moved to the opposing side is always:

*rubik distance - 1*

because to get from one side of the tile to the other (as the rubik distance is measured by) there will be one tile that will be counted twice.



Fig. 16, a 3i tile for example.

In Figure 16 it is shown that to get from the top-left to bottom-right of the tile 3i, to get from left to right 1 rubik distance is counted.

Then, since in a checkerboard arrangement, adjacent tiles are of opposite colours, for each set of two tiles, there will be a [Black|White] configuration.

Thus, if there are an even number of tiles, will be some sets with no tiles left over. Since a set always ends in the opposite colour, and when two sets are put together, since the end of the set is the opposite of the start of the set, the end of the chain of sets will always be the opposite of the start of the first set. Moreover, since an even number of tiles equate to an odd number of rubik distances, a pentomino with an odd number of rubik distances that



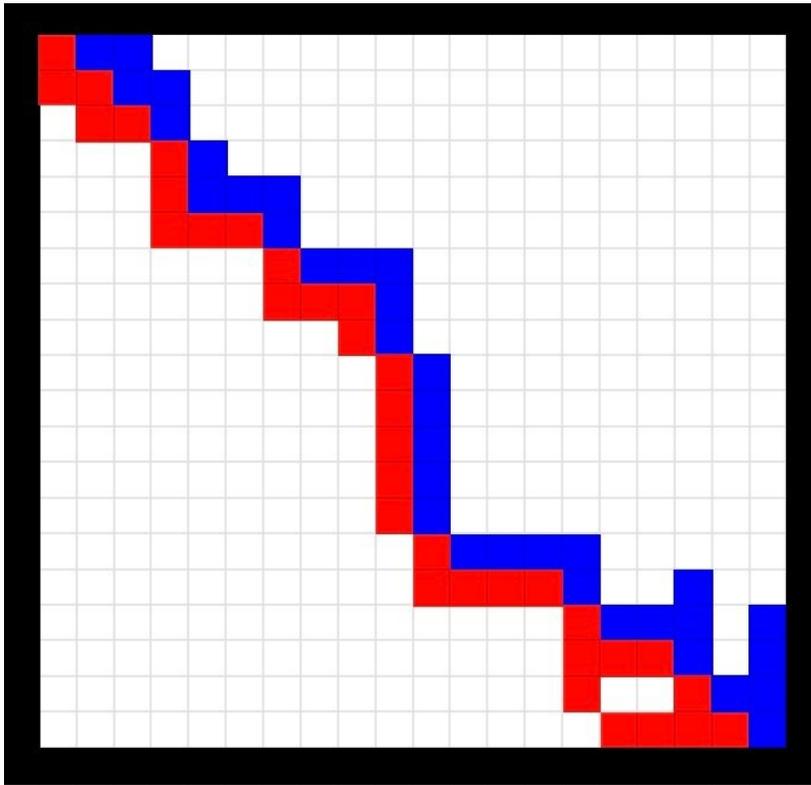


Fig. 19

### 5.3.3 Sealing off, conservation and blocks used.

Since we will spend 7 moves on expansion, the opponent would have tried their best to use as much of their own land as possible. Thus, there is no need for “sealing off” as we have already effectively sealed off much of their possible area by layering. Thus, we will continue with conservation for the rest of the game

Conservation will follow the same rules as tile consumption as mentioned in section 4.3. Since expansion is not affected by tile consumption, we will use pentominoes with the highest tile consumption for each category (rubik distance 5 and 6) to expand. The pentominoes, therefore, to use are 5i, 3L2, 4Z, 3Z, W, 4L1 (rubik distance 6) and X (rubik distance 4) for expansion.

The remaining pentominoes are as follows, and should be used in this order, from top to bottom:

Tiles	Tile Consumption
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3L1, R	16
C, 4T1, 3T2	15
4i, P	14
X	13
O, 3T1, 2Z2	12
3i	11
2L1	10
2i	8
1i	5

This is, therefore, the strategy for 2v2 blokus.

## 6. Conclusion

Once again, as aforementioned, our research questions are: What is the smallest square grid in which all pieces of tile area 4 and below can be placed, What are the most important tiles to play at certain times of the game if it is a 1v1v1v1 game and last but not least, What is the best strategy in a 2v2 game. For our first question, 11x11 was eventually the smallest square grid, derived through the usage of tiles and tile consumption to derive the answer. For our second question, 3 different phases of expansion, sealing off and conservation. We also derived which blocks were the best in each phase. Last but not least, the best strategy for 2v2 would be to layer your pieces with your teammates to separate the opponents, in order to divide and conquer.

## 7. Appendix

### Appendix A

Appendix A shows the table of all the units of measurement involved in the research of the project

Tile	Rubik D.	Blockage (Double/Single)	Tile C.
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O	4	0 / 12	12
1i	2	0 / 4	5
2i	3	0 / 8	8
3i	4	2 / 8	11
4i	5	4 / 8	14
5i	6	6 / 8	17
2L1	4	0 / 10	10
3L1	5	1 / 11	16
3L2	6	3 / 11	16
4L1	6	3 / 11	12
2Z2	5	0 / 12	12
3Z	6	2 / 12	15
4Z	6	2 / 12	15
3T1	4	1 / 10	12
3T2	5	3 / 10	15
4T1	5	3 / 10	15
R	5	1 / 12	16
W	6	0 / 14	14
X	4	0 / 12	13
P	5	1 / 12	14
C	5	2 / 10	15

## 8. Citations

Hart, E. (n.d.). Blokus Strategy. Retrieved July 30, 2019, from <http://blokusstrategy.com/>

Kershner, K. (2012, March 21). How Blokus Works. Retrieved July 30, 2019, from <https://entertainment.howstuffworks.com/leisure/brain-games/blokus2.htm>

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