Project Title: **<u>Minesweeper</u>**

Category 8: Mathematics

Group ID: 8-10

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1. Introduction

1.1 Description

The aim of our project is to find the most efficient way to complete a game of Minesweeper. A table with the tabulations of all the most common scenarios and solutions will also be created to assist the achieving of our aim.

1.2 How Minesweeper is played

Firstly, the player can choose where to place his starting move

This is an example of a scenario after a starting move (The move was the top right corner of the grid) :

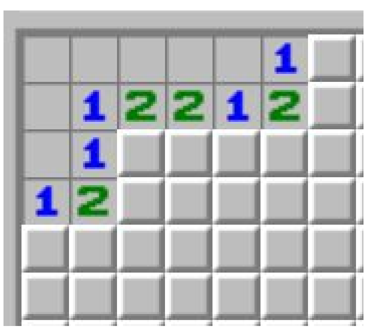

Figure 1a

The player can then place a flag at where he/she think the mine is:

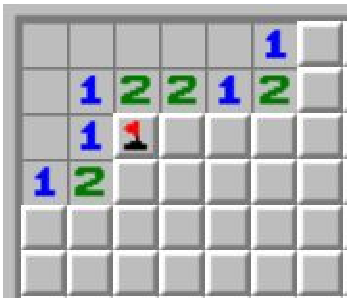

Figure 1b

There can be 2 scenarios that can happen next:

<u>1st scenario</u>

The grid with a flag has correctly been identified as a mine as another grid has been uncovered (The grid with the number ‘3’ inside):

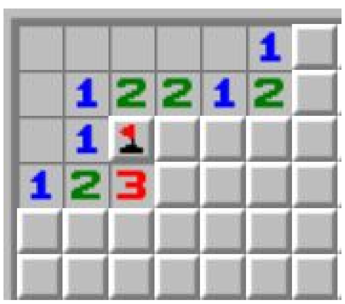

Figure 1c

The player can then continue the game by identifying the mines again

<u>2nd scenario</u>

The grid with the bomb with a cross was the grid where the player thought the mine was The mine was incorrectly identified and the game is over.

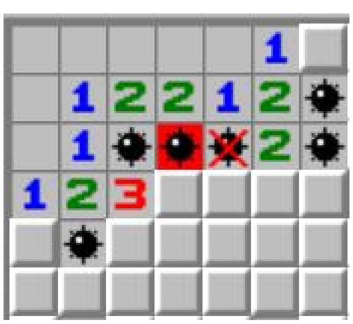

Figure 1d

Definitions:

Corner: The square next to 2 edges in the grid (3 squares around it)

Edge: The square next to only 1 edge in the grid (5 squares around it)

Center: The square that is not beside any edge in the grid (8 squares around it)

Patch: A group of squares where one square has 0 mines around it

Best way to start the game: The best way is for a move to have the highest chance to open a patch

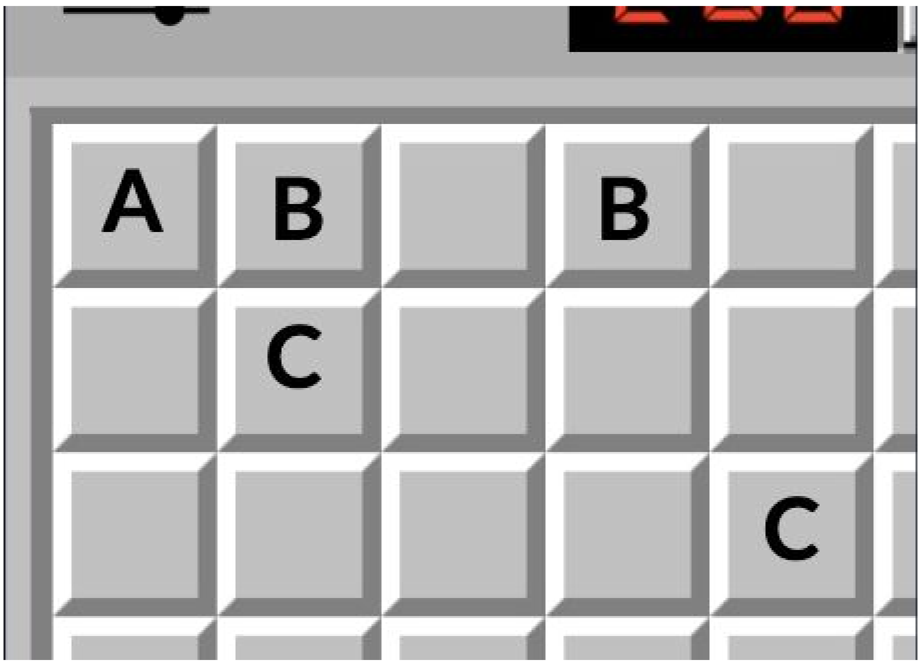

Figure 2

In this case, A represents a corner, B represents an edge, and C represents a centre.

1.3 Rationale, Objectives and Research Questions

1.3.1 Objectives

We have a few objectives for the project:

1. Assist players in finding the best way to start and end the game
2. Use tabulation to record instances where a certain common scenario appears
3. Find out a ‘best move’ when guessing is needed

1.3.2 Rationale

To improve ourselves and other players at Minesweeper and let us and them have a very good chance of winning the game through our research

1.3.3 Research questions

1. Is it better to start in the center, edge or corner?
2. How to guess as little as possible and when is it necessary?
3. How do you quickly calculate the probability of a square being a mine when guessing?

2. Literature Review

There is a website called ‘minesweeper.info’’ where there is a section named ‘Strategies and tips’. This section contains detailed strategies to simple situations contributed by many people. The same website also shows statistics for the best starting move for every mode of the game but does not explain the process of calculating the statistics.

More advanced research in this area of our project can be found in this paper titled “Some Minesweeper Configurations”, that discusses very complicated permutations of the game and also provide a method, which are “gates” that can be used to solve the permutations.

Some posts recorded on http://www.minesweeper.info/articles/DanielSilevitch-MinesweeperGuide-1992 mention the 1-2-1 and 1-2-2-1 patterns.
http://www.minesweeper.info/archive/BrianChuMinesweeper/website.html also discusses wall (1-1) and 1-2 patterns briefly.

3. Methodology and Study

3.1 Methodology

We used probability to calculate chances of starting and ending squares being mines and used tabulation to find common scenarios in the game. Further methodology will be explained in the results of the research questions themselves.

3.1.1 Probability for starting moves

We will use probability to calculate the chance that one square around the opening square is a mine. Then, we will calculate the chance of the next square being safe, assuming that the first square that we checked is safe. Lastly, repeat the step for all the remaining squares

Example

Probability of first square being safe:

$$\frac{479 - 99}{479}$$

The Minesweeper expert grid has 480 mines in the form of 16 by 30 and has 99 mines Thus, '479' would be the number of unopened squares and '99' is the number of mines remaining and to find the probability, we found the number of squares that could be safe (479-99) and divided it by the total number of unopened squares (479). Next, for the next square, we just deducted 479 by one as that would be the new number of unopened squares and recalculated the probability. We repeated the previous step for each square that was around the starting move and lastly, multiplied all the probabilities together to find the final probability of the starting move revealing a patch, which was our intended result.

3.1.2 Tabulation

We wanted to determine what patterns we common in the game so we could help players to identify them and solve them.

Firstly, we identified a few patterns that we thought were common and used tabulation to find out how many times the patterns, regardless of what form, would appear in a game to find out common patterns. We assigned one column for each game and counted all the times it appeared and recorded it in a table. We then combined all the numbers together throughout 50 games to find out the total number of times each pattern occurred and placed it in the last row of the table.

Example for pattern 1-2-1:

Game	Number of times it occurred
1	E.g. 12
2	E.g. 10
...	...
	Average number of times per game: e.g. 8

We then gave solutions to the patterns that we had identified as they turned out to be occur quite often during a game

3.1.3 Probability for moves needing guessing

We used a certain pattern that required guessing to use our method. Firstly, we used a mine counter and assumed the number of mines left in the grid. We then set up scenarios that could happen and solved those scenarios using probability. With these scenarios we can calculate the chance of a desired outcome by using combinatorics.

For example, if we have 2 different cases,

The probability of case 1 happening is just case1/(case1+case2)

Remaining squares would just be solved based on the number of mines that are left. (Note that this does not help solve any independently affected cases. E.g 50-50 cases.)

3.2 Results

All solutions are done in Expert mode.

3.2.1 Results to Research Question 1

A patch is defined in 1.2. There are 99 mines in Expert mode. We first calculate the probability of the first square surrounding the opening square being safe. We divide the number of safe squares in the grid by the number of unopened squares left in the grid

We get:

$$\frac{479-99}{479}$$

or 380/479. Next, we find the probability of the next surrounding square being safe. The total number of safe squares has decreased by 1, since the first surrounding square is safe.

We now have:

$$\frac{478-99}{478}$$

or 379/478. Similarly, the probability of the surrounding square being safe is:

$$\frac{378}{477}$$

Hence, the probability for a move in the corner to reveal a patch:

$$\frac{380\cdot 379\cdot 378}{479\cdot 478\cdot 477}$$

≈49.85%
or about 1 in 2

Similarly,

Probability for a move at the edge to reveal a patch:

$$\frac{380 \cdot 379 \cdot 378 \cdot 377 \cdot 376}{479 \cdot 478 \cdot 477 \cdot 476 \cdot 475}$$

≈31.25%
or about 3 in 10

Probability for a move in the center to reveal a patch:

$$\frac{380 \cdot 379 \cdot ... \cdot 374 \cdot 373}{479 \cdot 478 \cdot ... \cdot 473 \cdot 472}$$

≈15.45%
or about 3 in 20

Thus, we conclude that the best starting move is at the corner as it has the highest probability of opening a patch.

3.2.2 Results to Research Question 2

1-1

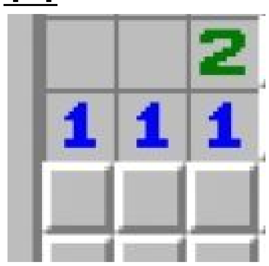

Figure 3.1a

In figure 3a, the “1” square next to the edge and the adjacent “1” square share 2 empty squares. To satisfy the former square, there is one (and only one) mine in the shared 2 squares. This means that the adjacent “1” square will also be accounted for and hence the bottom right square will be safe.

1-2

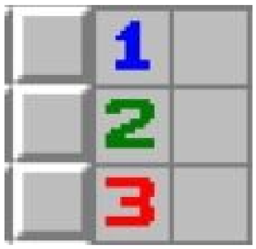

Figure 3.1b

In figure 3b, the square with number “2” shows that 2 out of the 3 squares are mines. If the mine is not next to “3” square, it has to be next to “1” and “2” squares, which is not possible as the “1” would have 2 mines around it. Thus one of the 2 mines has to next to the “3” square.

1-2-1

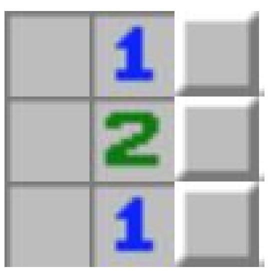

Figure 3.1c

In figure 3c, 2 out of the 3 empty squares are mines based on the “2” square. Suppose the mine is next to the “2” square, the other mine would have to be next to one of the “1” squares. Either way, one of the “1” squares would be surrounded by 2 mines, which is a contradiction. Thus the 2 mines are next to both “1” squares, while the square next to the “2” is safe.

1-2-2-1

Figure 3.1d

Similarly, in figure 3d, the only possible solution is one mine below each “2” square.

T pattern

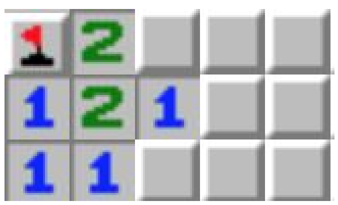

Figure 3.1e

In figure 3e, either one of the squares above the rightmost “1” square has to be a mine, so the squares on the right of said “1” square must be safe.

Occurrences in Expert games

Game	number of 1-1	number of 1-2	number of 1-2-1	number of 1-2-2-1
1	15	7	5	4
2	14	8	5	6
3	12	10	8	6
4	15	10	9	5
5	16	13	12	8
6	15	9	6	4
7	9	12	11	7
8	13	11	10	5
9	11	8	6	8
10	15	7	6	5

11	17	11	11	8
12	18	8	7	6
13	14	7	6	9
14	13	7	7	5
15	10	12	11	7
16	14	8	7	6
17	19	8	5	4
18	15	11	10	8
19	14	11	9	5
20	14	10	7	7
21	16	12	10	4
22	13	12	12	6
23	16	8	6	7
24	15	9	7	5
25	17	10	9	4
26	16	7	6	9
27	20	6	6	6
28	14	11	10	6
29	13	10	7	5
30	17	9	8	7
31	15	13	13	5
32	17	8	7	4
33	16	10	9	6
34	13	12	12	5
35	11	11	10	8
36	18	7	6	6
37	15	9	8	8
38	14	10	9	6
39	18	9	7	5
40	16	10	10	8
41	13	11	9	6
42	14	9	7	8
43	17	8	7	7
44	18	8	8	5

45	13	13	11	8
46	14	14	13	5
47	21	9	8	7
48	16	9	7	9
49	18	10	9	6
50	19	9	6	7

Average per game:

1-1: 15.12

1-2: 9.62

1-2-1: 8.3

1-2-2-1: 6

T pattern: 1 to 2

By showing the number of occurrences of each pattern, we know that all these are very common patterns and it would be very likely for a player to encounter such patterns, thus we have come up with solutions to them. The less common T pattern also shows up 1 to 2 times in a game.

3.2.3 Results to Research Question 3

The number of mines remaining can be used to our advantage.

When most of the board has already been solved, we can use the remaining mines to calculate probability of most of the squares. To do this, we can only work with mutually affected squares. This means that information in one square can be used to solve the remaining squares.

This means some situations can only be solved by guessing.

Examples:

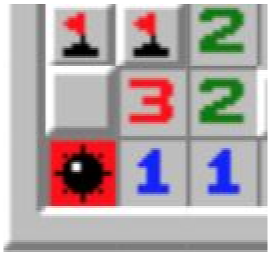

Figure 3.2a

There are no surrounding squares which can help solve the probability of these 2 remaining squares, thus it leaves the person to do a 50-50 guess

We will look at the following figure to as an example:

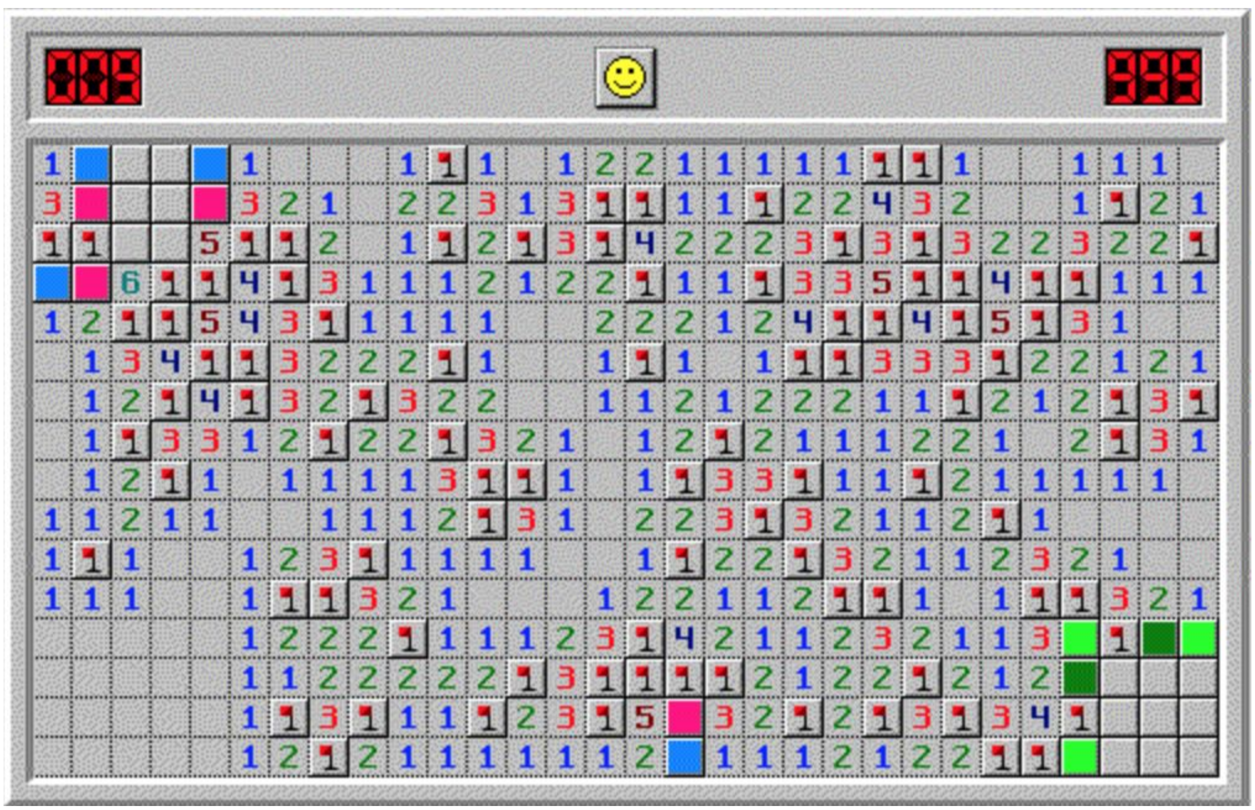

Figure 3.3

Light and dark green squares: mutually affected squares

Red and blue squares: Independently affected square

Looking at the bottom right corner of the board:

(Any coordinates in the working is represented in the figure below in terms of an (x,y) coordinate system)

	1	2	3	4	5	6
1	1	2	2	1	0	0
2	1	⚑	⚑	3	2	1
3	1	3		⚑		
4	1	2				
5	2	4	⚑			
6	⚑	⚑				

Figure 3.4a

Assume the rest of the board is solved. We will use the mine counter at the top.

It is quite obvious to see that the squares (3,3), (3,4), (3,6), (5,3) and (6,3) are affected by each other.

This means if we know one of these is a mine, we can determine the rest of the mines.

Thus we have 2 different situations for those squares.

Situation A:

	1	2	3	4	5	6
1	1	2	2	1	0	0
2	1	⚑	⚑	3	2	1
3	1	3	🚩	⚑	s	🚩
4	1	2	s			
5	2	4	⚑			
6	⚑	⚑	🚩			

Figure 3.4b

🚩: recently added flag

s: safe square

Situation B:

	1	2	3	4	5	6
1	1	2	2	1	0	0
2	1	⚑	⚑	3	2	1
3	1	3	s	⚑	⚑	s
4	1	2	⚑			
5	2	4	⚑			
6	⚑	⚑	s			

Figure 3.4c
⚑: recently added flag
s: safe square

Note that the group of squares from (4,4) to (6,6) (call this Group A) is not defined and there can be anywhere from 0 to 9 mines in it.
So, if there are 12 mines left, Situation A must be correct and all the squares in Group A are mines.
If there are 11 mines left and Situation A is correct, then 8 of the 9 squares in Group A are mines, so there are 9 possibilities. If Situation B is correct, then all 9 squares in Group A are mines, and there is 1 possibility. Therefore if there are 11 mines left, Situation A has a 9 ÷ 10 = 90% chance of occurring, while Situation B has a 1 ÷ 10 = 10% chance of occuring.
If there are 10 mines left and Situation A is correct, then 7 of the 9 squares in Group A are mines, so there are 9 choose 7 (or 9**C**7) = 36 possibilities. If Situation B is correct, then 8 of the 9 squares in Group A are mines and there are 9 possibilities. So if there are 10 mines left, the probability of Situation A happening is 36 ÷ 45 = 80% and the probability of Situation B happening is 9 ÷ 45 ≈ 20%.

Similarly, if there are 9 mines left and situation A is correct, there are 9**C**6 = 84 possibilities. If situation B is correct, there are 9**C**7 = 36 possibilities. The probability of Situation A is 84 ÷ 120 = 70% and the probability of Situation B happening is 36 ÷ 120 =

30%.

If there are x mines left in this particular situation were $x \leq 10$, the number of permutations of Situation A happening is 9**C**(x-3) and the number of permutations of situation B is 9**C**(x-8).

Obviously, this can be used in many different situations.
We can follow some steps which would lead to the probability being found:

1. Pick an unsolved area and list various outcomes of where the mines can be.
2. Find which is the more likely outcome
3. Subtract the number of mines left from the areas with guaranteed entered mines(e.g. A 50-50 position has definitely one mine)
4. Use the result to find the probability of the cases happening
5. The case with a lower probability of a mine would be the most desirable outcome.

4. Conclusion

Through this project, we successfully used probability and tabulation to help find ways to improve the chances of a player winning the game through our research. However, we had some limitations that prevented us from letting a player to have an improved chance of winning in every single game.
Firstly, the starting move that we calculated to be the best only had about 50% chance of being a patch. Thus, a player could still start at a corner and encounter a mine or just open one square. We cannot try to improve on this as Minesweeper is partially a game of luck but our research can also help the player to get a patch with the starting move in 50% of his/her games, which is also a good thing.
Similarly to the starting move, our calculations for the probability for the moves that require guessing also do not have a 100% chance of it not being a mine thus, a player might not always win the game but still has a good chance to do so. We also only provided one solution to the scenarios that required guessing and it might not work all the time.

Lastly, we only found scenarios that we thought were common to give solutions but there could be a lot more scenarios that a player could face during the game.
We can definitely extend our project by finding a lot more scenarios and giving solutions to them, while also thinking of a few more solutions to scenarios that require guessing, giving our project a lot more potential in the future.

5. References

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