Folding Math

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1 Introduction

There was a myth that paper cannot be folded in half for more than 7 times. However recently it had been proven that it all depends on the area and the thickness of the paper. According to Britney Gallivan, it all depends on when the thickness increases while the width/length decreases until when the thickness is larger than the smallest side. She even has the formulas $L=\pi t/6(2^n+4)(2^n-1)$ for when the paper is folded one way and $W=\pi t 2^{\frac{3(n-1)}{2}}$ for when the paper is folded in two directions simultaneously while using square papers.

1.1 Rationale

The formulas allow room for variation as there can be multiple shapes a paper can take and also different ways said paper can be folded.

1.2 Objective

1. To analyse the two formulae by Britney Gallivan.
2. To compare the two formulae by Britney Gallivan.
3. To work on the extensions for Britney Gallivan's formulae.
   (a) Final length and thickness is equal.
   (b) Least amount of paper length for n folds.
   (c) Most amount of paper length for n folds.

1.3 Research Questions

1. How do the formulae work?
2. How should the formulae be compared?
3. What is the range of the extensions?

1.4 Field of Math

1. Algebra
2. Geometry
3. The application of ratio

2 Terminology

Term	Explanation
Fold	When a whole piece of paper is folded in half along its side in order to double the amount of parallel layers without any tears created in the process.
Parallel Layer	Layers which are parallel and not bended.
Neutral Axis	A line or plane through a beam or plate connecting points at which no extension or compression occurs when it is bent.
Fold Indefinitely	If the next fold will result in the paper's height longer than its width and / or length, it is considered as cannot be folded anymore.

3 Literature Review

After Britney Gallivan has found her famous formula of folding paper, blog writers such as Chris Higgins has written blogs on the subject, showing other experiments of folding paper more than the originally thought 7 times made by people such as the Mythbusters and students at St. Mark's School in Southborough, Massachusetts. However they did not break the record however as the paper was not one singular piece of paper nor did it stay intact, which broke the rules required to make it a valid process of folding. Other writters such as Ashish on the scienceabc website brought about how folding paper more than 100 times may get you to the moon.

However it was a a blogger under the name ‘Udopern’ who wrote a page which explained the formulae that Britney had created. The page explains different approaches to how to calculate the minimum length needed for each fold.

The first method was to give the simple assumption that when a paper is folded in half, the length decreases in half while the thickness increases in the multiplication of 2. Hence for each fold L/T is divided by 4. Hence the formula is $L/T=4n$.

However in reality, when a paper is folded, a crease is created at the side, forming a semicircle shape which cannot be folded. The formula firstly focus on this problem, and multiple information can be found are that for every fold k, there are $2k-1$ sets of semicircles, where each semicircle have an individual length, which are start from $1T$ and increase by $2T$. Also, each set consists of and individual amount of semicircles, where the set containing the longest semicircle has one of its kind, while the second longest set has two, the third set has three and so on. Then a new set of sets are created labeled as fk,where fk consists of one of each semicircle up to the semicircle consisting of length $(2k-1)T$.

Hence $fk=T\cdot 4(k-1)2$ after simplifying. Then Fk is the total of all fks in the kth fold.

Since L is equal to the sum of the lengths of the parallel layers and the semicircles on the sides, then

$$L = 2k \times lk + Fk = 2k \times lk + \frac{(\frac{\pi}{2})T(2^{2k}-1)}{3}$$

'udopern' then continues to assume that *lk=fk*, where n is the desired amount of folds. This resulted in the formula $LT = 2^2n + 2 \times (2^2n - 1)$. However it was preferred that $lk = \frac{\pi}{2} \times T$ as it allows better simplification while still allowing *lk* to not to too long compared to T which would affect the amount of folds. In this situation, the formula $L = t \times 2n + 4(2n-1)6$ can be deprived upon simplification.

'udopern' also tried out situations for minimal and maximum gaps between ends of the paper, each resulting in their own formula, which are $\pi t(4^n - 1)/6$ and $\pi t(2^{n+1}+1)(2^{n+1}-1)6$.

However the blogger also admits that there are still limitations on the calculations, as all the formulas have not included neutral axis, which the the factor affecting the flexibility of the paper in a physical situation, which is why the minimalistic formula cannot be used.

4 Methodology and Results

4.1 Objective 1

For the first objective, we have decided that to analyze the two formulae, we should collect the value of L for as many values of n as possible. We first decided to do so using an app called

Desmos to draw a graph with y for L and W and x for n while t is one. In this way it will show the ratio of L to t. However since the formula has a y=z ^{x} where z is bigger than 1, meaning that the graph is exponential, hence the values were not observable as the graph turns into a vertical line. Hence we decided that Desmos was not a good tool to collect data.

Then, we tried to code a C++ program to calculated the results. Although the program was able to output the results in a way which can be read and copied, but C++ had another problem. As the program required the power function and the C++ code for it runs using the 'int' setting. This restricted the height of the value of which the outputs can be, so we were unable to find answers for values n that is bigger than 40. Not only so since the second formula involves the power to values that are not whole numbers the C++ problem which used 'int' settings were unable to provide accurate data for the second formula.

Hence instead we resorted to writing the problem in Scratch. In Scratch the program allows for accurate answers with the power to non-integers. Hence we were able to collect over 500 data for each formula before it reaches the capacity limit.

4.2 Objective 2

For the second objective, we calculated the the factor of the increase between all values of lengths required for a new fold, and we then calculated the average of the values.

For the first formula, the factor is around 3.984362976, while for the second formula, the factor is around 2.828427125.

4.3 Objective 3

For our third objective, we repeated the process of objective 1 and 2 on the three extensions that 'udopern' came up with. For the formulae calculating t=l, the factor of increase is 3.963327148, while for the minimalistic and maximum formula the factor of increase is around 3.972356433, except that in the maximum formula, it is exactly less than one more fold.

However we were not satisfied with just this, which is why we decided to create a few more formulae on our own.

Firstly we tried to create a formula to calculate how many folds are possible if we only know the length of the diagonal of the paper. The least diagonal length required for n folds would be to assume that the paper has no width, therefore we can use the minimalist formula: $L = \pi t/6(4^{\mathrm{n}} - 1)$. Hence one may assume that any length from $L = \pi t/6(4^{\mathrm{n}} - 1)$ to $L = \pi t/6(4^{n+1} - 1)$ is enough. However this is false as we can actually increase the width slightly, therefore increasing the diagonal length but the width is not long enough more any new folds. After calculations the correct range is $L = \frac{\pi}{6}t(4^{\mathrm{n}} - 1)$ to $L = \frac{\pi}{6}t \times \sqrt{(4^{\lfloor n/2 \rfloor} - 1) + 2^{n-\lfloor n/2 \rfloor} \times (4^{n-\lfloor n/2 \rfloor} - 1)}$ as the length of the diagonal of the paper for n folds.

We also tried to calculate a formula for the maximum amount of times you can fold an isoceles right triangle in half. We realised that this is actually quite similar to folding a paper one way as it also only requires one factor and the length decreases with a same factor everytime a fold is made and the curves still follow the same type of physics. The only difference is that the length decreases by a factor of $\sqrt{2}$ instead of 2. Therefore the final formula is $L = \frac{\pi}{6}T(4^n + 3 \times \sqrt{2} - 1)$

5 Conclusion

In this project, we have modified the formulae developed by Britney Gallivan to calculate the length needed to fold n times on a right-triangle and also calculate a diagonal length required to fold n times. We have also presented the mechanics in Britney's formulae to allow the users to understand more about Britney's formulae.

6 Limitation and Further Extension

There are still limitations in the formulae for folding paper, as it assumes that people are able to fold the paper perfectly, which can be difficult for the user, and also that the paper can be bended easily as long as it is long enough, as some types of paper may be too stiff to be folded. Also, as the thickness increases exponentially with each fold, therefore it also requires extremely accurate knowledge of the paper's initial thickness.

As for extensions, other than the formula which can calculate the length required to fold n times with a right triangle, there can also be formulae to calculate for the right angle triangle, there can still be formulae to be made for circles and also other equilateral shapes of paper.

7 Acknowledgement

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