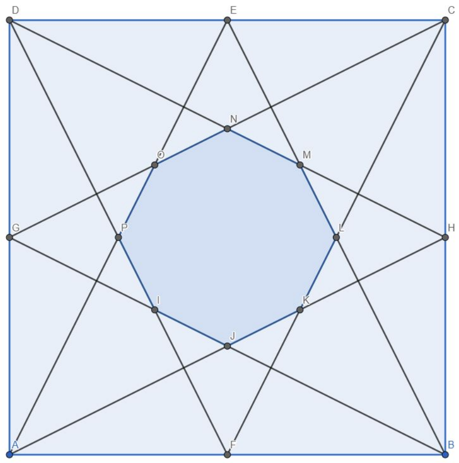
Rational Polygons (Group 8-26)



In this geometry problem, a line is drawn from each of the corners to the midpoints of the 2 sides that are adjacent to the opposite corner. The aim is to find out the ratio of the area of the octagon to the area of the square or other quadrilaterals. Since there is already a past project about the ratio when the outer polygon is a quadrilateral, my project aims to extend this to polygons with more than 4 sides, such as pentagons and hexagons. The objective is to find out the relationship between the area of the n-sided polygons (where n>4) and the area of the 2n-sided polygons in them, and the general trend as n increases.

The research problems are:

1. To find out the relationship between the area of the areas of n-sided *regular* polygons and the area of the 2n-sided polygons in them
2. To find out the relationship between the area of the areas of n-sided *irregular* polygons and the area of the 2n-sided polygons in them
3. To find out the trend of the relationship between the area of the areas of n-sided polygons and the area of the 2n-sided polygons in them

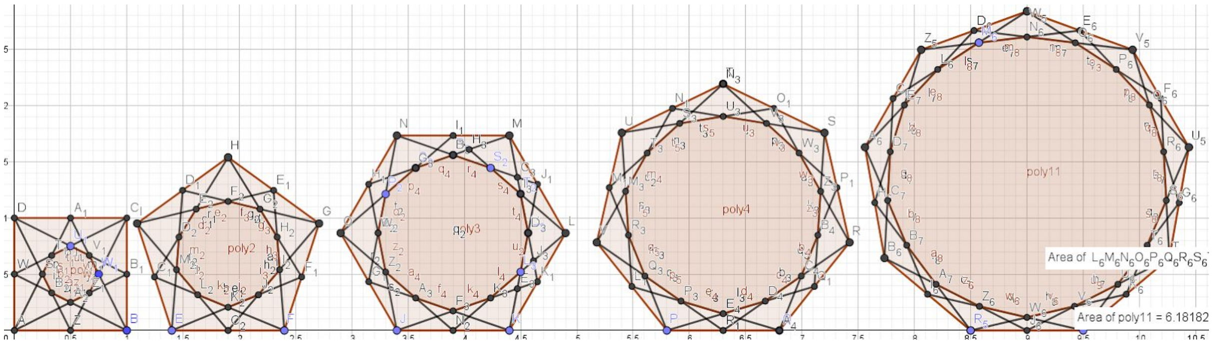
Literature review - Don't be square

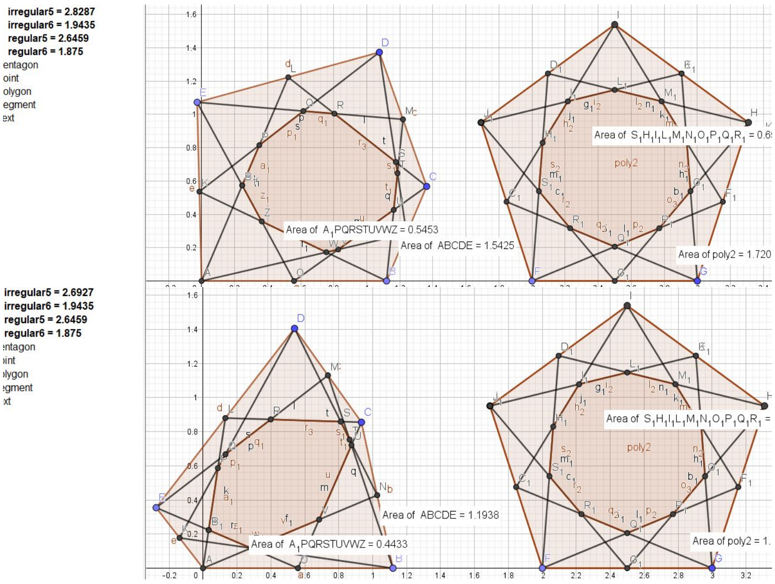
In this past project, it was found that the ratio of the octagon to the square is 1:6.
For other quadrilaterals, if the quadrilateral can be split into 4 parts with equal shape and area, the ratio of the area of the octagon to the area of the quadrilateral is 1:6.

Methodology

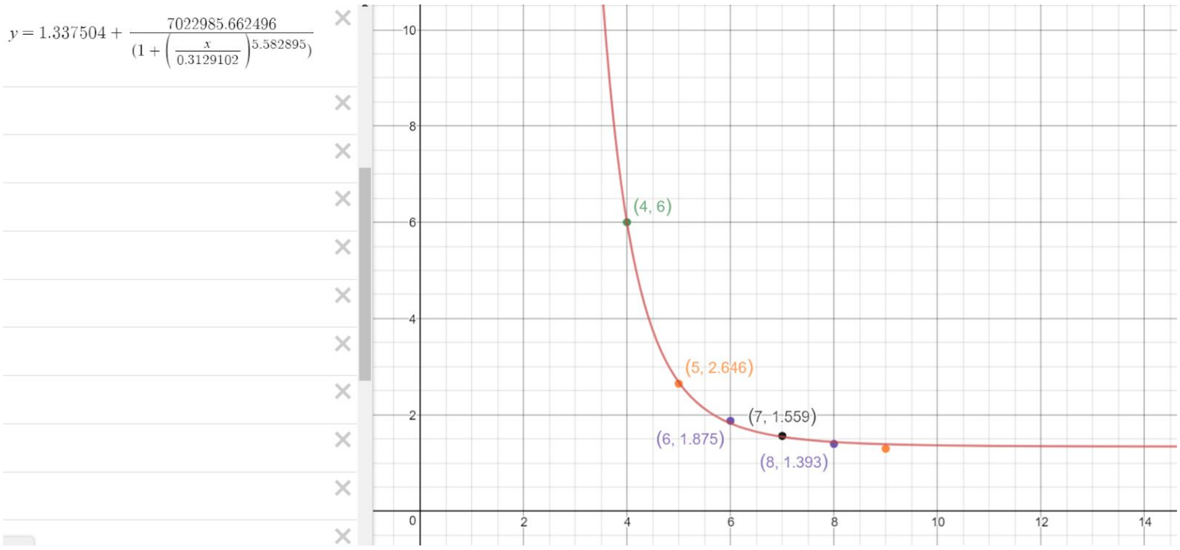
To create the 2n-sided polygons, a line would be drawn from every corner to the midpoints of the sides that are adjacent to the adjacent sides. Manually calculating the vertices of both outer and inner polygons is very tedious so I used tools like GeoGebra to calculate the areas of the polygons. GeoGebra also allowed me to more easily see how the ratio changes as each vertex of the irregular polygon is moved.

Number of sides	4	5	6	7	8	9
$Area_{outer}:Area_{inner}$	6.000000000	2.645898034	1.874941563	1.558842991	1.393398282	1.294173594





The ratio of the area of the outer polygon to the inner polygon is always greater for irregular polygons than for regular polygons.
However, the inner polygon could not be constructed if any of the angles of the outer polygon is a reflex angle as some of the lines would no longer intersect.



As the number of sides approaches ∞, the ratio of the inner polygon to the outer polygon approaches 1.

Possible extensions

Find out a formula for calculating the ratio of the areas of the regular polygons
Find out a formula for calculating the ratio of the areas of the irregular polygons

References

Teoh, J. J., Tan, Z. W., Foo, S. H., & Loo, S. Y. H. (n.d.). Don't be square. Retrieved May 5, 2018, from https://dontbesquarecat9.weebly.com/