Tan Ler 2i323, Asher Ng 2i220, Calen Tang 2i203, Loh Chi Wen 2i315

House of Math V2

Contents

Introduction

This project is a continuation of a similar project from last year, called House of Math. The objectives of that project were to find the distance of coverage (distance between landing point and roof edge) for various roof shapes and angles and investigate the effects of using different tiling styles.

The assumptions made for that project are:

1. For triangular roofs the hypotenuse is 5m, with varying angles of 30, 45 and 60 degrees. The building height for all structures is 10m.
2. The raindrop has a diameter of 6mm and a terminal velocity of 10m/s. It also has an initial velocity of 0m/s.

The path of the raindrop is shown in the below diagram. The raindrop starts at the tip of the roof, flows down the hypotenuse and off the roof until it reaches the ground. The distance is then calculated and this is what we refer to as the distance of coverage. (Fig. 1)

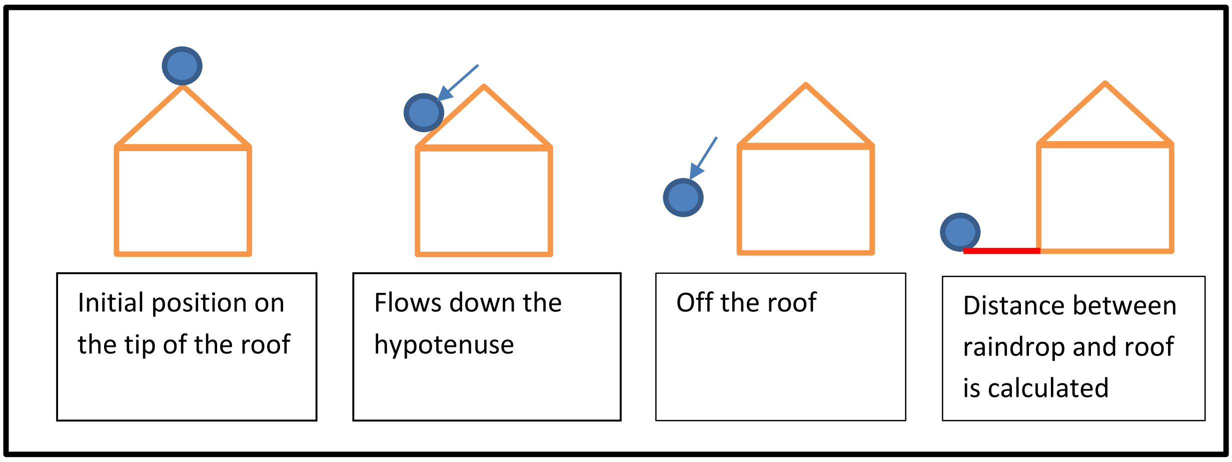

Fig 1: Path of a raindrop down a roof.

The results obtained for the triangular roofs are:

30° Roof - 7.25m

45° Roof - 6.41m

60° Roof - 4.80m

The distance of coverage provided by the dome was found to be 2.42m. A tiling style was also investigated, which gave the distance of coverage of 10m. The results seemed to be inaccurate due to the exclusion of friction.

Upon further investigation, the best angle for a triangular roof was also concluded to be 26 degrees after a graph was plotted for roof angles ranging from 0 to 90 degrees.

We decided to expand on this project as it has some interesting real life applications such as finding the optimal position to install a drainage system. We also want to learn more about the physics and math behind such real world problems. Lastly we would like to address some limitations in last year's project due to some assumptions being made.

We hope to improve on this project by using more true to life dimensions for the structure. 2.1m and 1.5m are stated by the LTA to be the minimum dimensions for a sheltered walkway, and thus those are the dimensions that we will be using. Another problem was that as friction was not included, the raindrop accelerated too fast on the tiled roof, which may lead to the large distance. We also would like to account for more factors which includes air resistance to make the results more precise.

Objectives

We want to tackle some issues with last year's project by including friction into calculations, factoring in varying raindrop sizes and finding a generalised equation for the problems.

Research Questions

1. Calculate rainwater runoff with inclusion of friction
2. Investigate how varying raindrop sizes affect runoff
3. Create a generalised formula which will give the distance of coverage when raindrop size and angle of roof are given

Literature Review

Raindrop - Speed

(Elert, n.d.)

States the average terminal velocity for certain sizes of raindrops.

Slopes - Inclined Planes

("Inclined Planes," n.d.)

Explains the forces acting on an object on an inclined plane and its movement when under the influence of gravity.

Average Raindrop Size - Singapore

(Åsen & Gibbins, 2002)

A study done on the average raindrop size in Singapore. There is a graph that shows the number density of raindrops of a certain size, which we base our raindrop diameter off.

Coefficient of Friction

("Coefficient of Friction," n.d.)

States that the coefficient of friction of ceramic tiles is 0.42 DCOF, which we used in our friction calculations.

Terminal Velocity of Fall for Water Droplets in Stagnant Air

(Gunn & Kinzer, 1949)

Shows the terminal velocity for water droplets in stagnant air.

Methodology

1. Find out the formulae and variables involved in each research question
2. Use specific numbers and input them into the various variables
3. Observe patterns and trends
4. Find out the optimal solutions of the problems.

The field of math that we will be covering in this project includes algebra, geometry, and mathematical physics.

Results

Terminology

F - Force

F_{grav} – Gravity

$F_{parallel}$ – Gravitational force parallel to roof

F_{norm} – Gravitational force perpendicular to roof

$F_{friction}$ – Friction

F_{net} – Net force

v - Velocity

v_{term} – Terminal velocity

v_{roof} – Velocity of the raindrop leaving the roof

v_{vert} – Vertical component of v_{roof}

v_{hori} – Horizontal component of v_{roof}

T - Time

T_{roof} – Time taken to reach the edge of roof

T_{term} – Time taken to reach terminal

T_{rem} – Remaining time

T_{total} – Total time

D - Distance

D_{rem} – Remaining distance

D_{cov} – Distance of coverage

Assumptions

Structure Dimensions (height by width):

2.1m by 1.5m

Roof:

A right-angled triangle with a variable angle.

Raindrop:

Shape - Spherical

Initial Velocity - 0m/s

Dynamic coefficient of friction (DCOF) of tile: 0.42

Research Question 1: Calculate rainwater runoff with inclusion of friction

The raindrop diameter is assumed to be 2mm

The angle of the roof is assumed to be 45 degrees.

The dimensions of the roof are given as follows:

Opposite: 0.75m

Adjacent: 0.75m

Hypotenuse: 1.06m

$Raindrop\ Volume$

$= \frac{4}{3} \times (0.001m)^3 \times \pi$

$= (4.19 \times 10^{-9})m^3$

$Raindrop\ Mass$

$= 1000kg/m^3 \times (4.19 \times 10^{-9})m^3$

$= (4.19 \times 10^{-6})kg$

F_{grav}

$= (4.19 \times 10^{-6})kg \times 9.8m/s^2$

$= (4.11 \times 10^{-5})N$

$F_{parallel}$

$= (4.11 \times 10^{-5})N \times \sin 45$

$= (2.90 \times 10^{-5})N$

F_{norm}

$= (4.11 \times 10^{-5})N \times \cos 45$

$= (2.90 \times 10^{-5})N$

$F_{friction}$

$= 0.42 \times (2.90 \times 10^{-5})N$

$= (1.22 \times 10^{-5})N$

F_{net}

$= F_{parallel} - F_{friction}$

$= (2.90 \times 10^{-5})N - (1.22 \times 10^{-5})N$

$= (1.68 \times 10^{-5})N$

$Net\ acceleration$

$= (1.68 \times 10^{-5})\ N \div (4.19 \times 10^{-6})\ kg$

$= 4.02m/s^2$

Following that, the terminal velocity of the raindrop had to be obtained. Fig. 2 displays the results that Gunn and Kinzer had found experimentally. (Gunn and Kinzer, 1949)

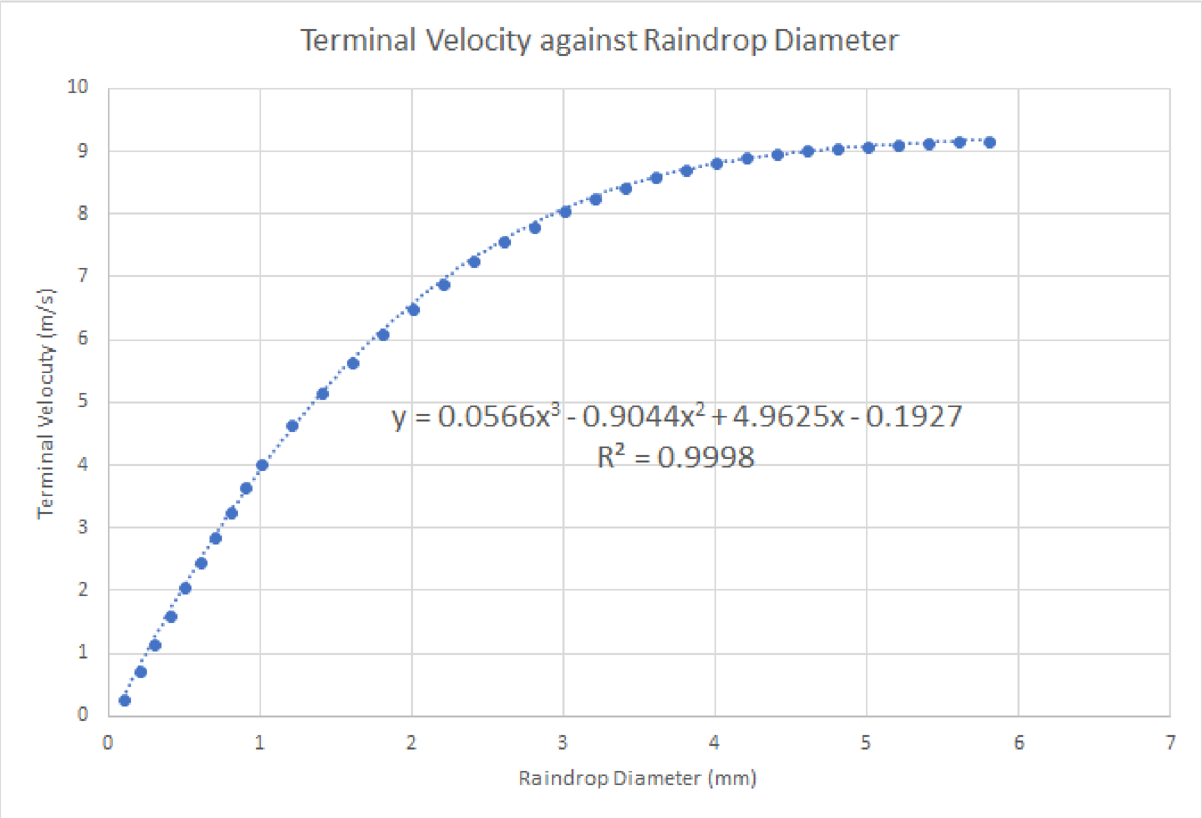

Fig. 2: Plotted graph of terminal velocity against raindrop diameter

Using a polynomial graph of the 3rd degree, the trendline shown on the graph was found through regression analysis. The formula is

$v_{term} = 0.0566x^3 - 0.9044x^2 + 4.9625x - 0.1927$

where x is equal to the raindrop diameter in millimetres. However, this is only valid when $0.1 \leq x \leq 5.8$.

In this case, $v_{term} = 6.56m/s$.

T_{roof}

$= \sqrt{1.06m \times 2 \div 4.02m/s^2}$

$= 0.726s$

V_{roof}

$= 4.02m/s^2 \times 0.726s$

$= 2.92m/s$

V_{vert}

$= 2.92m/s \times \sin 45$

$= 2.06m/s$

V_{hor}

$= 2.92m/s \times \cos 45$

$= 2.06m/s$

T_{term}

$= (6.56m/s - 2.06m/s) \div 9.8m/s^2$

$= 0.460s$

D_{rem}

$= 2.1m - [2.06m/s \times 0.460s + 0.5 \times 9.8m/s^2 \times (0.460s)^2]$

$= 0.117m$

T_{rem}

$= 0.117m \div 6.56m/s$

$= 0.0178s$

T_{total}

$= 0.460s \; + \; 0.0178s$

$= 0.478s$

D_{cov}

$= 2.06m/s \times 0.478s$

$= \mathbf{0.985m}$

Thus, for a raindrop of 2mm diameter, the distance of coverage is **0.985m**.

<u>Research Problem 2: Investigate how varying raindrop sizes affect runoff</u>

We took different raindrop sizes, from 1mm to 2mm diameter each with a 0.05mm difference from each other.

Inputting these diameters into the above steps, a graph of raindrop diameter against distance of coverage was plotted. (Fig. 3)

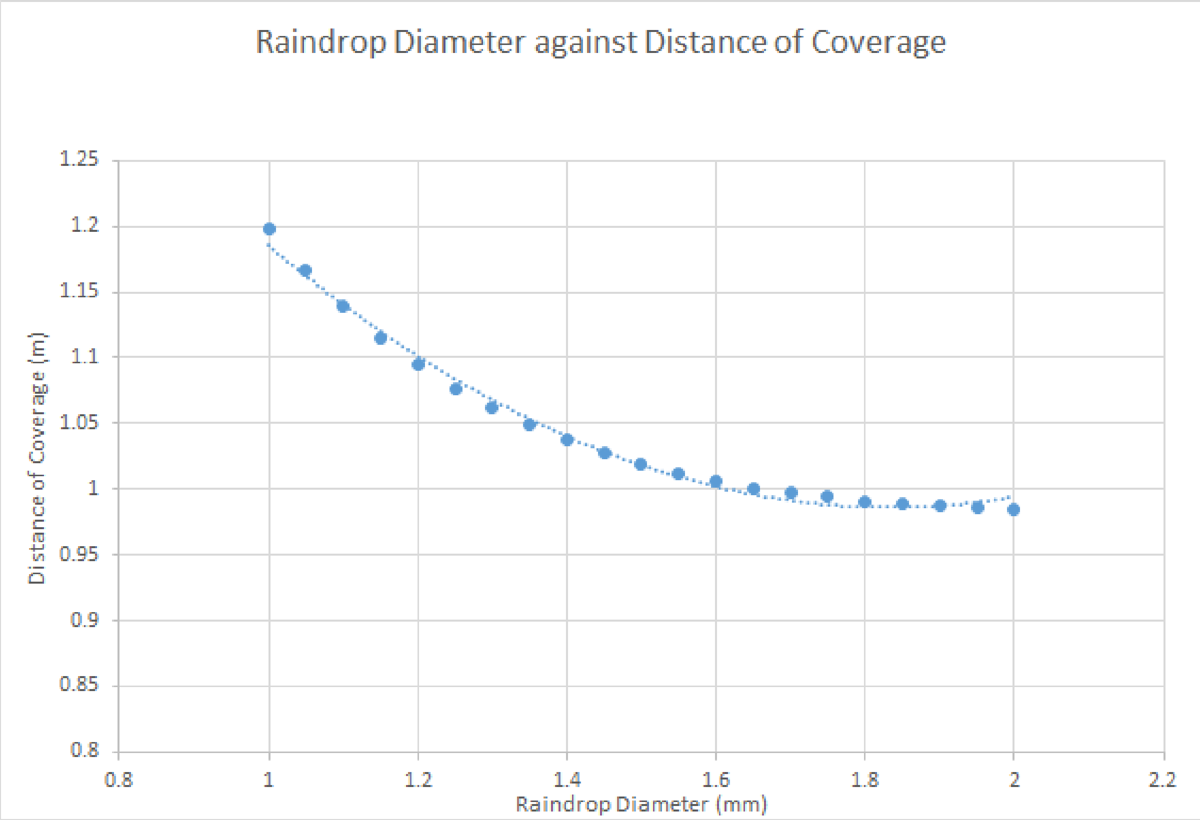

Fig. 3: Plotted graph of raindrop diameter against distance of coverage

Based on regression analysis, the distance of coverage can be estimated using this quadratic formula.

$D_{cov} = 0.286x^2 - 1.0494x + 1.9488$

Where x is the raindrop diameter in mm and 1 ≤ x ≤ 2.

<u>Research Problem 3: Create a generalised formula which will give the distance of coverage when raindrop size and angle of roof are given</u>

Variables used:

x – Diameter of raindrop

θ – Angle of roof

The formula is given below:

$$k_1\left[2.1 - \frac{3k_2^2 - 2k_1k_2 - k_1^2}{19.6k_2}\right]$$

$$Where$$

$$k_1 = \sqrt{2.12 \div 9.8(\sin\theta - \cos\theta \times 0.42)} \times 9.8\sin\theta \times 0.42(\sin\theta - \cos\theta)$$

$$k_2 = 0.0566x^3 - 0.9044x^2 + 4.9625x - 0.1927$$

Conclusion

1. For a raindrop of 2mm diameter and a roof angle of 45 degrees, the distance of coverage is **<u>0.985m</u>**.
2. Distance of coverage is affected by raindrop diameter in the following quadratic function:

$$D_{cov} = 0.286x^2 - 1.0494x + 1.9488$$

Where x is the raindrop diameter in mm and 1 ≤ x ≤ 2.

3. The following is the generalised equation for raindrop diameters of 1mm to 2mm:

x – Diameter of raindrop

θ – Angle of roof

$$k_1\left[2.1 - \frac{3k_2^2 - 2k_1k_2 - k_1^2}{19.6k_2}\right]$$

$$Where$$

$$k_1 = \sqrt{2.12 \div 9.8(\sin\theta - \cos\theta \times 0.42)} \times 9.8\sin\theta \times 0.42(\sin\theta - \cos\theta)$$

$$k_2 = 0.0566x^3 - 0.9044x^2 + 4.9625x - 0.1927$$

Applications

The results of this project can be used to provide a good estimate on the positioning of drainage systems near shelters.

Thus, this brings about more efficient rainwater catchment.

Limitations

Due to the fact that drag coefficient is calculated experimentally, we were unable to use the formula for terminal velocity, and had to instead use pre-determined values. This may have caused the calculations to be slightly inaccurate.

The regression model for terminal velocity is limited to the domain:

$0.1 \leq x \leq 5.8$

It is worth pointing out that raindrops rarely exceed 6 mm in diameter because they become unstable when larger than this and break up during their fall. (Hayden et al., 2018)

The formula from Research Problem 3 only applies to raindrop sizes within a range of 1mm to 2mm. This is a relatively small range and may not be sufficient.

The results were also inaccurate when terminal velocity is not obtained before hitting the ground.

Further Extensions

We would like to account for cases when terminal velocity is not obtained before hitting the ground. With the current generalised formula, it is always assumed that terminal velocity has been reached before the raindrop reaches the ground. However, in cases where this does not happen, the distance of coverage given by the formula will be negative.

We would also want to calculate distance of coverage with inclusion of varying initial velocities and drag coefficient. These are other factors which will result in the final distance of coverage being more accurate.

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