Written Report

1. Introduction of our project

- Our project is about using cars to find the length of trains, or to put it in another way, using cars as a new unit for measuring the length of trains. This is to better estimate the number of passengers to accommodate and the length of trains.
- Cars are at least 1 metre long, a convenient unit to measure trains.
- **Objectives:**
    1. Based on original procedure and project details, find a formula to calculate k in terms of n.
       (Experimental and for comparison with formula based on objective 1; both objectives 2 and 3 are separate.)
    2. With cars of up to lengths of ‘5’, find a formula to calculate k in terms of n.
    3. With a train width of ‘2’, find a formula to calculate k in terms of n.
- Research problems:
    1. Number of cars whether of length ‘1’ or ‘2’(or ‘3’-‘5’ based on objective 2), needed to measure length of train.
    2. Ways to fit cars of length ‘2’ in a train of width ‘2’ based on objective 3.
- Scope of study:
    1. Fibonacci
    2. Formulating

Project details

- Independent variable: Length of train, n
- Dependent variable: k (number of trains formed)
- Procedure
    1. Different sets of train based on order and placement of cars, k. Eg. 1-1-2, 2-1-1.

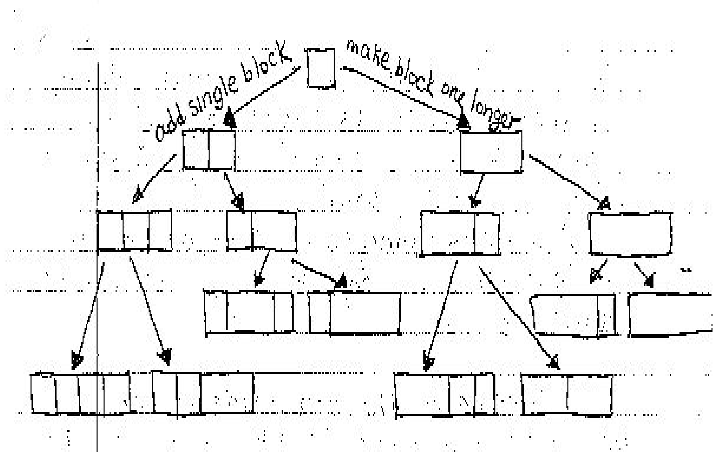
2. Cars of same length but placed in different orders will not be recorded. Eg. 2-2-2, 2-2-2.

- Controlled variable : Length of cars, “1” and “2”
- Others: Objective 2 (Cars of lengths of up to ‘5’); Objective 3 (Train of width ‘2’)

2. Literature review

Megan Lojek. (1999). [Paperwork]. Available: http://www2.edc.org/makingmath/studentWork/trains/lojek/trains_lojek1.asp . Retrieved February, 1999.

Megan analysed k for n=(1~5) and used the tree method whereby you start of with a single block and keep on adding one with each left arrow or increasing it by one block’s length with each right arrow until you are able to draw out all train sets of length “n”. This method produces every train with length “n+1” with endless possibilities for both sides. She also marked a table in which the output could be replaced with a power of 2 and the input is “n”. She found out that each exponent was one value less than its input and eventually arrived at the formula: 2^{n-1}



(n) LENGTH OF TRAIN	# OF TRAINS PRODUCED		
1	1	>1	$= 2^0$
2	2	>2	$= 2^1$
3	4	>4	$= 2^2$
4	8	>8	$= 2^3$
5	16	>16	$= 2^4$
6	32	>32	$= 2^5$
7	64	>64	$= 2^6$
8	128		$= 2^7$

Jackie Ou. (1999). *Program in Mathematics for Young Scientists.* [Paperwork]. Available: http://www2.edc.org/makingmath/studentWork/trains/ou/trains_ou1.asp. Retrieved 1999.

Jackie has analysed how n number of 1s are lined up and noted that: he was able to either include a "+" or exclude one between 2 consecutive 1s, thus having 2 choices of in or out "+" for each (n-1) space; he was able to place (n-1) number of "+" in the (n-1) spaces and know that there are [(n-1)/0+(n-1)/1+(n-1)/2+...+(n-1)/(n-1)] ways to partition n; he found out that he must choose (k-1) number of the (n-1) spaces to put "+" between, thus concluding there is 2^{n+1} ways to partition n and coming up with a theorem that there are (n-1)/(k-1) trains that can be made with "k" number of cars (Where order matters). He then came up with a few conjectures on partitioning “n”.

n	1	2	3	4	5	6	7	8	9	10
p(n)	1	2	3	5	7	11	15	22	30	42

Conjectures:

* There are $\left[\frac{n}{2}\right]+1$ ways to partition n into sums using only summands 1 and 2.

* In general, there are $\left[\frac{n}{x}\right]+1$? ways to partition n into sums using only and x as summands? where $x > 1$.

* (N into exactly (k+1) partitions) + (N into partitions of size at most k) = (N into parts of size at most (k+1)).

  i.e. $P_{k+1}(n) + {}_{k}P(n) = {}_{k+1}P(n)$.

* $P_k(n) = P_k(n-k) + P_{k-1}(n-1)$.

* The number of partitions with an odd numbers of parts is equal to the number of partitions with distinct summands.

* The number of partitions into k part is equal to the number of partitions of size at most k.

Erik Elwood. (1999). *Program in Mathematics for Young Scientists.*

[Paperwork]. Available: http://www2.edc.org/makingmath/studentWork/trains/elwood/trains_elwood1.asp. Retrieved 1999

Erik focused on the definition of partitioning and its functions. His investigation focused on how many ways the non-positive integer “n” can be partitioned where either the order for k matters or does not matter.

Based on the investigations the above authors have carried out, they focused on the partitioning of non-positive integer “n” to find the formula whereby they are able to find the number of trains formed in terms of “n”.

However, there are endless possibilities and variables to this question in which they have only scratched the surface of. There is no other result to compare with their findings.

Our group planned to explore some of these possibilities such as width of train and length of cars and draw conclusions as to the comparison between the original formula and formula from special conditions.

3. Methodology

- Excel sheets: Listing down of n and k for data recording
- Google sheets: Constructing of graph for clearer interpretation and comparison
- Paperwork: Formulating relationship between n and k

Results

- <u>Objective 1:</u>
    1. Fibonacci sequence
    2. Quadratic equation
- <u>Objective 2:</u>
    1. No formula.
    2. Alternative solution: Have cars of lengths up to infinity
- <u>Objective 3:</u>
    1. $(n-1)^{th}k*3+1=n^{th}$ term of k

Objective 1

Length of train\No. of cars	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
1	1															**1**
2	1	1														**2**
3		2	1													**3**
4		1	3	1												**5**
5			3	4	1											**8**
6			1	6	5	1										**13**
7				4	10	6	1									**21**
8				1	10	15	7	1								**34**
9					5	20	21	8	1							**55**
10					1	15	35	28	9	1						**89**
11						6	35	51	36	10	1					**144**
12						1	21	80	74	45	11	1				**233**
13							7	51	136	115	55	12	1			**377**
14							1	28	136	210	155	66	13	1		**610**
15								8	74	262	345	205	78	14	1	**987**
	No. of trains formed															

Objective 2:

Length of train\No. of cars	1	2	3	4	5	6	7	8	Total
1	1								1
2	1	1							2
3	1	2	1						4
4	1	3	3	1					8
5	1	4	6	4	1				16
6		5	10	10	5	1			31
7		6	15	20	15	4	1		61
8		7	21	31	31	21	7	1	119
	Number of trains formed								

Objective 3:

Length of train\No. of cars	1	2	3	4	5	6	7	8	Total
1	1	1							2
2		2	4	1					7
3			3	11	7	1			22
4				5	24	27	10	1	67
	Number of trains formed								

4. Conclusion

Formulae for all 3 objectives to be compared

- Objective 1: increment is in relation to fibonacci sequence
- Formula: $\mathbf{F_n=(1/\sqrt{5})[(1+\sqrt{5})/2]^{n+1}+-(1/\sqrt{5})[(1-\sqrt{5})/2]^{n+1}}$
- Objective 2: With only cars of lengths of up to ‘5’, increment from train of length ‘6’ became inconsistent

- Formula for alternative solution: $\mathbf{2^{n-1}}$
- Objective 3: For n=1, 2 trains will be formed. For the subsequent values of n, [(n-1)th k]*3+1
- Formula: **$2*3^{n-1}$+[(3^{n-1}/2)Round down to nearest whole number]**

Future extensions:

1. Increase in width of train
2. Limited no. of cars to build train of _ length