Go Bingo!

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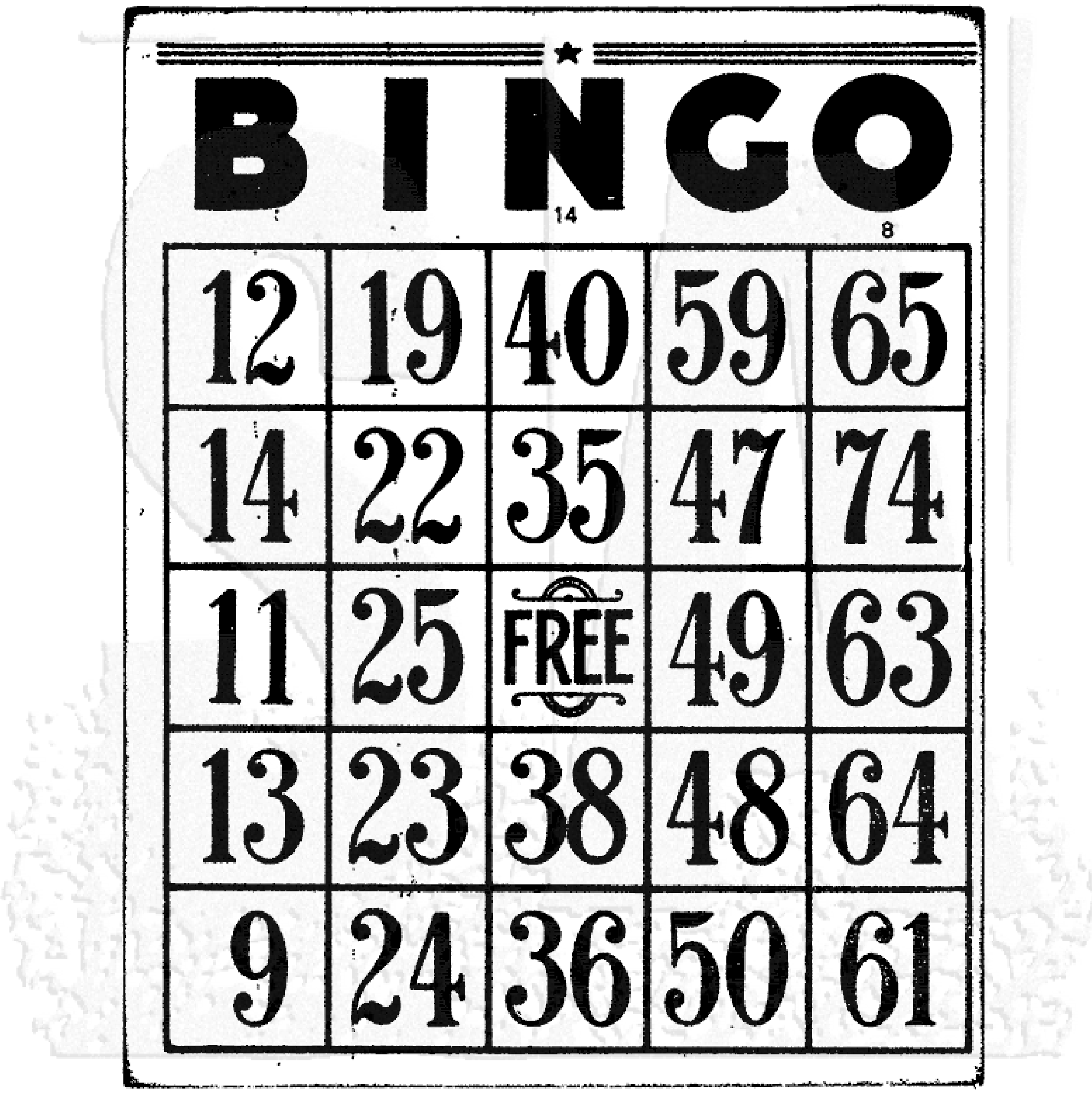
1 Introduction

Bingo is a game of chance where numbers are called out, and crossed off when they match the numbers on your card. The first player to cross off numbers in a row, column and diagonal calls out “BINGO!” and wins. In addition, the Free-Cell in the middle will be crossed off by default at the start.

This project aims to analyse Bingo and calculate the chances of winning concerning the game.



1.1 Rationale

We think that Bingo games have many variations, it also lends itself well to permutations and combinations and is simply fascinating at the same time!

1.2 Objectives

1. To find out the general formula for having a winner within x moves with y players in the Free-cell bingo.
2. To find out the minimum amount of moves needed, so that an amount of chance of winning on that move is met.
3. To find out the changes in the winning chances of the Free-cell bingo, if the column of the number that is chosen has to be observed.

1.3 Research Questions

1. How should the probabilities of winning on each move be added in the Free-cell bingo?
2. What is the pattern of the increase in probability of winning in the Free-cell bingo after every move?
3. How can this change be compared to the results of which when the column of the number in the Free-cell bingo do not need to be observed?

1.4 Scope(s) of study

1. Combinations
2. Possibilities

2 Literature Review

According to the article, ’Bingo Games: Turning Student Intuitions into Investigations in Probability and Number Sense’ in the magazine ’The Mathematics Teachers, Volume 93’, someone will get a Bingo in 7 out of 10 games played. However, certain numbers are fixed (eg. It is always 20 moves). So, we will like to find a general formula for having a winner within x moves within y people.

Also, according to Durango Bill, the key to calculating Bingo statistics is the Single Board Cumulative Probability of getting Bingo on or before the “n^{th}" number is called (Durango Bill, 2011). However, it is too common, every website on probability of Bingo shows this. So, we would like to find a pattern on the increase instead and explain the pattern.

3 Study and methodology

For the first objective, we used the formula to win with and without the Free-cell from the article ’Bingo Games: Turning Student Intuitions into Investigations in Probability and Number Sense’ in the magazine ’The Mathematics Teachers, Volume 93’, which is

$$4\frac{\binom{x}{4}}{\binom{75}{4}}$$

with the free cell and

$$8\frac{\binom{x}{5}}{\binom{75}{5}}$$

without the free cell. As $\binom{x}{y} = \frac{x!}{(x-y)!y!}$, we simplify the formulas to be

$$4\frac{71!4!x!}{75!(x-4)!}$$

with the free cell and

$$8\frac{70!5!x!}{75!(x-5)!}$$

without the free cell.

As these are only the chances to win exactly on the x^{th} move, we add them with the probabilities to win on the previous moves until move 4 (with the free cell) and move 5 (without the free cell).

After adding, the probability to win with x moves with the free cell will be

$$\frac{71!4!4(1+5!+\frac{6!}{2!}+\frac{7!}{3!}+...+\frac{x!}{(x-4)!})}{75!}$$

and

$$\frac{70!5!8(1+6!+\frac{7!}{2!}+\frac{8!}{3!}+...+\frac{x!}{(x-5)!})}{75!}$$

without the free cell.

If we add these two together, we will get something like this

$$\frac{70!4!4y[71(1+5!+\frac{6!}{2!}+...+\frac{x!}{(x-4)!})+10(1+6!+\frac{7!}{2!}+...+\frac{x!}{(x-5)!}]}{75!}$$

where x is the number of moves and y is the number of players.

For the second objective, we break down how the probability is calculated, to find a pattern.

For the probability with the free cell in 4 moves, you need to get 4 numbers out of 75 on the first draw, hence $\frac{4}{75}$. Once one of the numbers you need is drawn, you need 3 numbers out of 74 on the second draw, hence $\frac{3}{74}$. And so on with $\frac{2}{73}$ and $\frac{1}{72}$ Then you multiply them together and also by 4 as there are 4 possible rows with the free cell.

For 5 moves, it is the same but with 1 extra move. There are 4 places to fail, before the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} draw. There is only one extra move, so we use n multichoose k concept. n, in this case, is 4 and k is 1, which is the number of moves minus 4. n multichoose k = $\binom{(n+k-1)}{k}$ = $\binom{(x-1)}{(x-4)}$ = $\binom{4}{1}$ = 4 (in this case). Therefore, there are 4 scenarios (Note: For multiple scenarios, the probability for each scenario is the same)

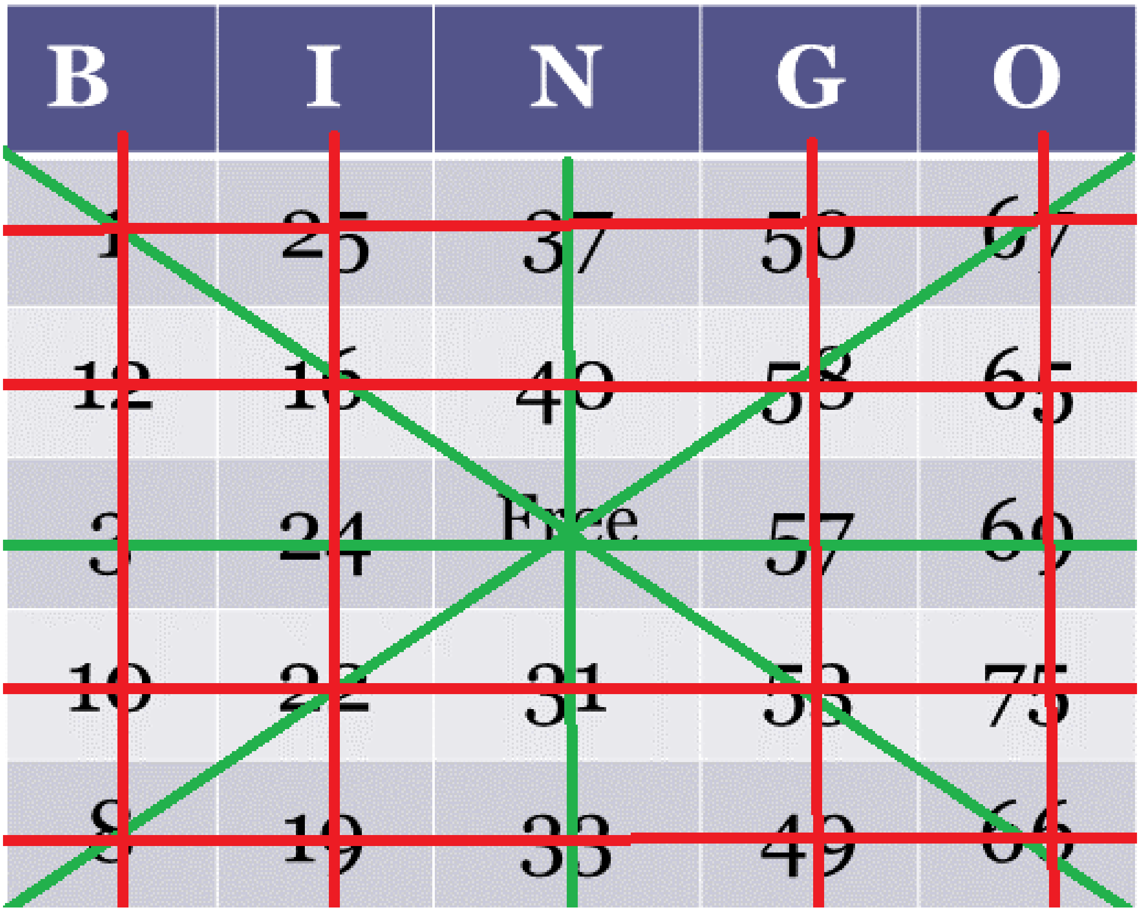
For the probability of winning without the free cell in 5 moves, it is

$$\frac{5}{75} \times \frac{4}{74} \times \frac{3}{73} \times \frac{2}{72} \times \frac{1}{71}$$

For 6 moves, there is 1 extra move and 5 places to fail, before the 1st, 2nd, 3rd, 4^{th} and 5^{th} successful draw. Again, we use the n multichoose k concept. n is 5 while k is 1. 5 multichoose 1 = $\binom{(x-1)}{(x-5)}$ = $\binom{5}{1}$ = 5 $\Rightarrow$ There are 5 scenarios.

Hence the pattern is the number of scenarios in each winning move. We can use this formula to clearly see the pattern and find the probability of winning on x^{th} move, to match the probability required to be met

$$\frac{70!24 \times [\binom{4}{0} + \binom{5}{1} + ... + \binom{(x-1)}{(x-5)}] + 284 \times [\binom{3}{0} + \binom{4}{1} + ... + \binom{(x-1)}{(x-4)}]}{75!}$$



There are 4 different lines to win with the help of the free cell and there are 8 different lines to win without the help of the free cell. The green lines show the way to win with the free cell while the red lines show the way to win without the free cell.

For the third objective, we use the basic formula for the first objective and have hypothesized 3 different ways(with the chances of winning of the fourth move as reference):

$$\frac{71!4!}{75!} or \frac{71!4!}{75!5^5} or \frac{371!4!}{375!}$$

The first method is made on the point that there are 5 columns, the chance is decreased by 80%. However, int this case the chance of winning on the x^{th} round(without observing the columns) is equal to winning on the $(1.5x)^{th}$ round(when observing the columns) since $\sqrt{5} = 1.5$ (2 s.f.), which is unrealistic, as there are 5 times the amount of possible values.

The second method is because it is assumed that the chance is divided by 5 for each number picked, but

$$\frac{71!4!5!}{75!5^5} > \frac{71!4!6!}{75!5^6}$$

which is obviously is a paradox as the chance of winning on the 6^{th} move is obviously greater than the chance of winning on the 5^{th} move.

The third method is because it is considered that there are $75 \times 5 = 375$ values, creating a new equation on the chances of winning on the x^{th} move:

$$\frac{371!4!x!}{375!}$$

Hence, we can conclude that the third one is correct. We can now find the formula for the last objective. With simple addition, it can soon be found that the formula for the last objective is

$$\frac{370!4!4y[371(1+5!+\frac{6!}{2!}+...+\frac{x!}{(x-4)!})+10(1+6!+\frac{7!}{2!}+...+\frac{x!}{(x-5)!}]}{375!}$$

where y represents the number of players and x is the number of moves.

4 Conclusion

From our working, we have found out the chance of winning in bingo is very small within the 4^{th} and 5^{th} move however the chance will increase immensely as more moves are taken. Also, when we apply additional rules in the game, the chance of winning can also decrease immensely.

4.1 Limitations

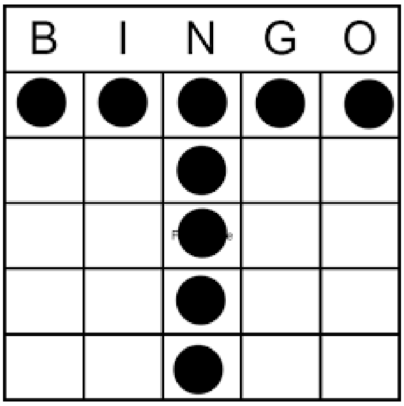
Tedious equations are involved within the formulas.

4.2 Possible Extensions

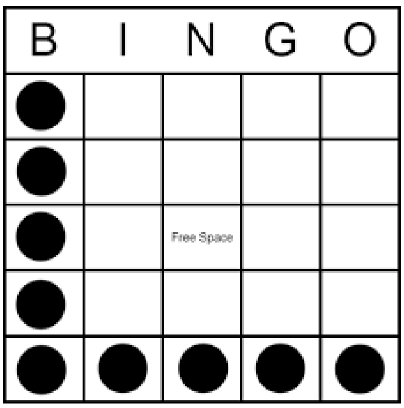
Here are some possible extensions or alternate play styles for Bingo:

4.2.1 Alternate Play Styles

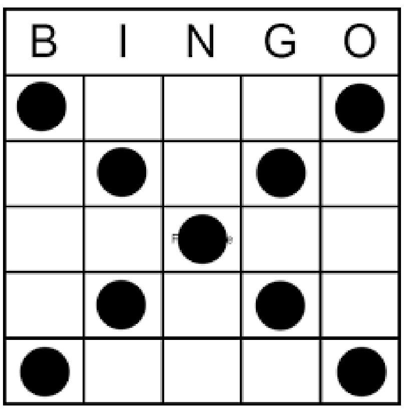
- T-Bingo



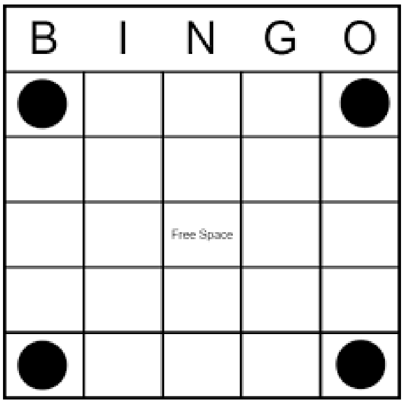
- L-Bingo



- X-Bingo



- Four-Corners



4.2.2 New ideas

Alternating column Bingo:

A Bingo game where the column is to be observed every two rounds instead of every round.

5 References

Here are our references:

Belfast Telegraph (2012) An introduction to bingo. Retrieved March 16, 2018, from: https://www.belfasttelegraph.co.uk/life/features/an-introduction-to-bingo-28750306.html

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